Regression approach to Portfolio Optimization

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Abstract

This study introduces a convenient approach to portfolio optimization by integrating regression and machine learning techniques into the traditional Mean-Variance Optimization (MVO) framework. We establish a connection between MVO and Ordinary Least Squares (OLS) regression, redefining asset allocation through a regression perspective. The methodology includes generating target returns for regression analysis and applying machine learning algorithms for enhanced portfolio management. This approach not only reinterprets portfolio optimization but also show-cases the potential of predictive models in dynamic financial markets.

Keywords: *Mean-Variance Optimization (MVO), Regression, Predictability, Machine Learning.*
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[1] Introduction

In the dynamic and unpredictable realm of financial markets, the quest for optimal asset allocation remains a pivotal challenge that compels continuous innovation in financial methodologies. Traditional approaches like Mean−Variance Optimization (MVO) in Markowits (1952) have long provided foundational strategies for balancing risk and return, not only in the stock market, but also bond market (Lee & Kim 2019). However, the static nature of these traditional models often fails to keep pace with the complexities and rapid evolutions observed in today's financial markets. Highly dependent on accurate estimates of asset returns and covariance matrices, the MVO model is prone to significant estimation errors in limited samples, which can substantially deviate from expected performance and adversely affect out−of−sample portfolio performance (Michaud 1989; Best & Grauer 1991). Green & Hollifield (1992) observed that inaccurate estimations of input parameters in constructing mean−variance efficient portfolios often result in extreme weights.

With the advancement of financial theory and empirical research, numerous studies have explored ways to mitigate the limitations of traditional MVO models. A significant approach has been the integration of regularization norms within the Markowitz optimization framework. Some studies have addressed estimation errors by regularizing statistical estimates (Ledoit & Wolf 2003a; Ledoit & Wolf 2003b; Garlappi et al. 2007), while others have focused on the regularization of portfolio weights (Brodie et al. 2009; DeMiguel et al. 2009b; Fan et al. 2012; Fastrich et al. 2015). They indicate that constraining weights is akin to reducing the sample estimation of co−variance matrices. Recent studies on portfolio optimization, such as those by Kremer et al. (2020) and Zhao et al. (2021), have employed various norm constraints and regularization techniques to enhance out−of−sample performance and stabilize asset weights. Meanwhile, advances in machine learning have opened new pathways for enhancing asset prediction accuracy and achieving superior investment returns (Kim et al. 2018; Gu et al. 2020; Nagel 2021). Ban et al. (2018) have optimized portfolios by incorporating performance−based regularization (PBR) and cross−validation techniques, and in Eckstein & Kupper (2021), a comprehensive mechanism to penalize incorrect predictions is demonstrated, extending beyond traditional L1 and L2 regularizations.

Our research diverges from previous studies that have focused primarily on imposing regularization constraints within the Mean−Variance Optimization (MVO) framework. We re−envision MVO through the lens of regression analysis, introducing a novel methodology that not only aligns with the theoretical foundations of traditional portfolio optimization but also leverages the robustness and adaptability of regression techniques. Our exploration begins by highlighting the inherent similarities between the solutions derived from MVO and those obtained through linear re−

gression, drawing upon insights from Hansen & Richard (1987). We advance this concept by proposing a methodology that treats traditional portfolio optimization as a special case of ridge regression. This method enhances the stability of parameter estimates and the generalizability of the model across diverse datasets, which not only provides fresh insights for addressing the challenges associated with MVO but also enriches the literature on portfolio-regularization. As demonstrated by its strong out-of-sample performance, our regression-based approach significantly broadens the application scope of MVO, enhancing its flexibility and suitability for navigating the uncertainties inherent in dynamic financial markets. While complex and diverse machine learning algorithms excel in capturing non-linear interactions (Gu et al. 2020), our methodology is straightforward yet effective, demonstrating substantial applicability in real-world scenarios.

The paper proceeds as follows: In Section 2, we present the analytical framework that forms the basis of our methodology, emphasizing the synergies between traditional portfolio theory and regression analysis. Section 3 provides a description of the data. Section 4 investigates the performance of portfolios constructed by our new approach compared to other methods. Section 5 presents our concluding thoughts.

[2] Analytical Framework

2.1 MVO and OLS: similarity

The similarity between the solution for weights in Mean-Variance Optimization (MVO) and the Ordinary Least Squares (OLS) regression estimates can be illustrated by comparing their respective formulas.

In MVO, we maximize the utility function $w^T\mu - \frac{\gamma}{2}w^T\Sigma w$ where μ is a weighting vector such as expected excess returns over risk free rate and Σ is a weighting matrix such as the covariance matrix for the excess returns. The solution for the optimal weights w to risky assets is given by:

$$\hat{w} = \gamma\Sigma^{-1}\mu$$

In OLS regression, the estimate of the regression coefficients β_{OLS} is:

$$y = r^T\beta + \epsilon$$

$$\beta_{OLS} = E(rr^T)^{-1}E(ry)$$

where r is a vector of independent variables and y is a dependent variable.

In MVO, μ and Σ are the expected value and the covariance matrix of asset returns. In OLS, $E(rr^T)^{-1}$ is the inverse of the expected value of the matrix product of the independent variable matrix with itself.

The inverse of the covariance matrix in MVO is multiplied by the vector of expected returns μ. In OLS, the inverse matrix is multiplied by $E(r^Ty)$, which is the expected value of the product of the transposed independent variable matrix and the dependent variable vector.

The γ in MVO represents the investor's risk aversion, scaling the investment according to the investor's tolerance for risk. By adjusting γ , one can adjust the allocation to a riskless asset, i.e., capital market line. There is no direct analog to γ in the OLS formula, as OLS aims to find the best linear unbiased estimator without a risk adjustment factor. MVO aims to balance return and risk in a portfolio, while OLS seeks to minimize the difference between observed and predicted values (residuals) in a regression model.

Both the MVO optimal weight solution $\hat{w}$ and the OLS regression coefficients β_{OLS} involve inverting a matrix that characterizes the relationships between different variables (covariance matrix in MVO, and the expected value of the matrix product rr^T in OLS). They then multiply this inverse matrix by a vector representing returns (expected returns in MVO, and the expected product of the independent variables with the dependent variable in OLS).

When we set $y = 1$ (regressing returns to one), the similarity becomes clearer (Hansen & Richard 1987): $E(r) = \mu$ and $\Sigma = E(\tilde{r}\tilde{r}^T) = E(rr^T) - \mu\mu^T$ where $\tilde{r} = r - \mu$. When $\mu = 0$ or r is demeaned, OLS (Ordinary Least Squares) solutions are the same as MVO (Mean-Variance Optimization) solutions (Brandt & SantaClara 2006; Britten-Jones 1999), although we have to plug in the mean of $r(u)$, which in turn should be estimated. Note that the simple ridge estimator is $\beta_{ridge} = (Err^T + \lambda I)^{-1} E(ry)$. Therefore, MVO can be considered a special case of ridge regression $(\lambda = -\mu\mu^T)$.

2.2 Portfolio allocation using ridge regression

To rewrite the linear (ridge) estimator:

$$\beta_{OLS} = (\Sigma + \lambda_1 \mu\mu^T)^{-1} E(ry) = (E(rr^T) - \lambda_2 \mu\mu^T)^{-1} E(ry) \quad (1)$$

In MVO, $\lambda_1 = 0$ (or $\lambda_2 = 1$) and $y = 1$. The following discussion explores alternatives to the classical MVO, originally formulated by (Markowits 1952), by framing these methods within the context of linear regression.

Instead of setting $y = 1$ as in traditional MVO, we use a target return as the dependent variable, $y = r_{target}$. For instance, the target return can be the sum of a benchmark return and an alpha, which can be either a constant or a random variable. A more aggressive target can be $r_{target} = \{r_{benchmark}, r_{min}\}$, which means that the portfolio should be constructed to follow a benchmark return, except in cases of significant negative returns for risk management purposes.

[3] Data

The universe of this study comprises global stock and bond indices sourced from FnGuide, encompassing both emerging and developed markets. Table 1 provides an overview of the universe of assets. The stock indices include those from 11 countries and region (Panel A). The bond indices are derived from the Asia−Pacific, Europe, South Korea, and the United States (Panel B). The sample period spans from January 2001 to December 2023, resulting in 276 monthly return observations across 18 assets (Table 2).

Table 1

Overview of the universe assets.

Name	Description	Country/Region
Panel A: Stock indices		
AORD	ASX All Ordinaries	Australia
BSESN	BSE Sensex	India
BVSP	Bovespa Index	Brazil
FTSE	FTSE 100 Index	United Kingdom
HSI	Hang Seng Index	Hong Kong
IXIC	NASDAQ Composite Index	United States
JKSE	Jakarta Composite Index	Indonesia
KS11	KOSPI Index	South Korea
N225	Nikkei 225 Index	Japan
STI	Straits Times Index	Singapore
000001.SS	Shanghai Stock Exchange Composite Index	China
Panel B: Bond indices		
APTR	Bloomberg Asian Pacific Aggregate Total Return Index	Asian Pacific
EUTR	Bloomberg EuroAgg Total Return Index	Europe
KCB	KIS Composite Corporate Bond Index	South Korea
KTB	KIS Korean Treasury Bond Index	South Korea
USCB	Bloomberg US Corporate Total Return Index	United States
USTB	Bloomberg US Treasury Total Return Index	United States
USHY	Bloomberg US Corporate High Yield Total Return Index	United States

Table 2
Descriptive statistics of the asset returns.

Panel A: Stock indices	AORD	BSESN	BVSP	FTSE	HSI	IXIC	JKSE	KS11	N225	STI	000001.SS
Observations	276	276	276	276	276	276	276	276	276	276	276
Mean (%)	0.028	0.684	0.432	−0.139	−0.172	0.393	0.72	0.345	−0.062	0.008	−0.025
Std. Dev.	0.04	0.06	0.066	0.039	0.056	0.053	0.053	0.054	0.054	0.045	0.065
Min(%)	−23.545	−32	−38.273	−21.928	−34.9	−25.188	−38.179	−32.266	−36.069	−31.859	−26.325
1st Quartile(%)	−1.971	−2.299	−3.174	−2.288	−3.209	−2.347	−1.688	−2.276	−3.015	−2.311	−3.483
Median(%)	0.649	0.785	0.505	0.657	0.345	0.989	1.253	0.885	0.383	0.481	0.241
3rd Quartile(%)	2.483	4.56	4.902	2.274	3.201	3.917	3.645	3.547	3.478	2.463	3.446
Max(%)	9.734	21.554	18.754	11.778	14.47	11.624	13.916	11.739	15.227	15.79	26.344

Panel B: Bond indices	APTR	EUTR	KCB	KTB	USCB	USTB	USHY
Observations	276	276	276	276	276	276	276
Mean (%)	0.17	0.264	0.295	0.267	0.31	0.213	0.478
Std. Dev.	0.006	0.011	0.011	0.005	0.016	0.012	0.024
Min(%)	−2.674	−4.078	−3.756	−1.449	−7.325	−3.415	−18.473
1st Quartile(%)	−0.125	−0.369	−0.248	0.004	−0.492	−0.604	−0.349
Median(%)	0.197	0.374	0.31	0.249	0.315	0.164	0.65
3rd Quartile(%)	0.549	0.876	0.85	0.529	1.166	0.962	1.55
Max(%)	1.999	4.138	5.099	3.341	5.984	3.448	7.208

[4] Empirical analysis

4.1 Strategy development

According to Britten−Jones (1999), the co−efficients in the regression approach represent the weights of risky assets in the portfolio. We performed ridge regression of the time series of the target returns on the time series of the assets in the universe to determine the portfolio weights. Considering the short sale constraint applied to all strategies, assets with negative coefficients in the ridge regression are assigned zero weights. A simple equal−weighted portfo−lio (EW−S), as described by Low et al. (2016), is constructed by distributing weights equally across the portfolio at the start of the sampling period and maintaining these weights through−out the investment horizon. The equal−weight portfolio in this study (EW−R) integrates both the regression approach and the simple equal−weight approach, with only assets pos−sessing positive regression coefficients being equally weighted in the portfolio construction.

To ensure the consistency of the results, short selling is prohibited in the mean−variance portfolio and the minimum variance portfolio. Additionally, L2 regularization is incorporated into these two approaches to prevent extreme weight values. Table 3 outlines the portfolio selection strategies. The S&P 500 is used as the benchmark return, with the target return defined as the maximum value between the S&P 500 return and zero.

Table 3
Portfolio strategies applied in the empirical analysis.

Strategy	Description
REG	Regression approach portfolio
EW-R	Equal weight portfolio excludes negative coefficient assets
EW-S	Simple equal weight portfolio (1/N)
MVSC	Mean-variance portfolio with short sales constraint and L2 regularization
MINC	Minimum variance portfolio with short sales constraint and L2 regularization

In this study, we employ rolling windows of both 180 and 120 months for regression analysis to estimate the weight matrix. We report results for M = 180 months in the sections that follow, while results for M = 120 months are presented in the supplementary material. Specifically, we apply ridge regression with various penalty parameters to the training sample, spanning from t-181 to t-2, to estimate weights for the validation set in month t-1. The model is finalized by selecting the parameter that yields the highest return in the validation portfolio. These optimized portfolio weights are then applied to the test set in month t to evaluate the out-of-sample return performance.

4.2 Portfolio rebalancing and terminal wealth

We proceed to evaluate portfolio rebalancing while accounting for the transaction costs associated with each strategy. The holding period is set to one month, and the average turnover is calculated in accordance with the methodology outlined by DeMiguel et al. (2009):

$$Average\ turnover = \frac{1}{n-S}\sum_{t=1}^{n-S}\sum_{i=1}^{N}\left(\left|\hat{w}_{i,t+1,k} - \hat{w}_{i,t,k}\right|\right)$$
(2)

where $\hat{w}_{i,t,k}$ is the portfolio weight for asset i at time t using portfolio strategy k . n is the total month of dataset. Sis the sample window, which is one month longer than the training window M. N is the number of assets.

Table 4 provides a summary of the descriptive statistics concerning portfolio turnover within a one-month holding period. Notably, the REG (ridge regression) portfolio demonstrates the highest and most pronounced turnover, averaging 0.510. This observation suggests that the REG portfolio undergoes more frequent and significant rebalancing in comparison to other strategies.

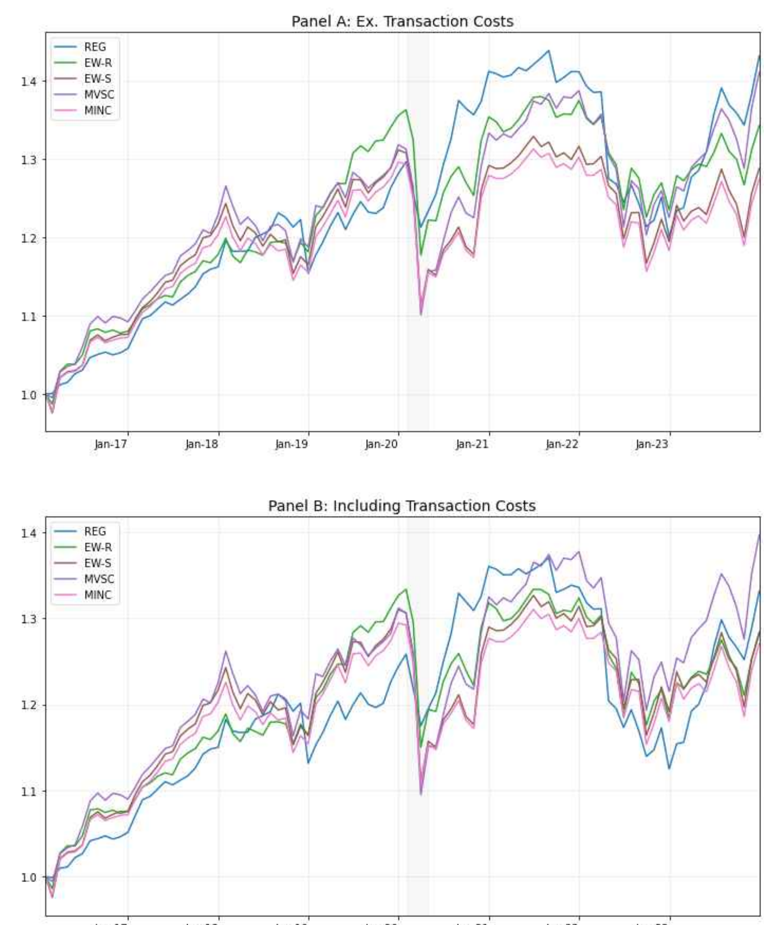
Table 4

Descriptive statistics of portfolio turnover. M = 180 months.

	N	Mean	Std	Min	Max
REG	95	0.510	0.526	0.010	1.320
EW−R	95	0.329	0.290	0.007	0.866
EW−S	95	0.021	0.010	0.000	0.075
MVSC	95	0.072	0.078	0.026	0.751
MINC	95	0.022	0.011	0.008	0.083

Figure 1

Cumulative return of the different strategies based on an initial investment of $1, using a rolling window of size M = 180 monthly observations. The shaded period corresponds to the crisis period. Panel A excluding transaction costs, Panel B including transaction costs of 15 basis points per transaction.



Our analysis compares portfolio selection strategies by assessing their performance through both terminal wealth and risk−adjusted performance measures. Figure 1 displays the cumulative returns achieved by each strategy, with and without transaction costsof 15 basis points, which is comparable to prior studies (Low et al., 2016; Sleire et al., 2022).

The COVID−19 recession is widely recog− nized as one of the most significant economic downturns in history. Beginning in February 2020, major stock market indices experienced a notable decline, continuing until April 2020 when markets began to recover. Although the REG strategy was initially slower to adapt in the early stages of the out−of−sample period, it demonstrated significant resilience during the crisis. Figure 1 shows that, whether trans− action costs are excluded, while other strat− egies suffered substantial losses during the COVID−19 recession, the REG strategy not only minimized drawdowns but also achieved the highest cumulative returns by the end of the recession. This demonstrates the REG method− ology's effectiveness in managing asset corre− lations and mitigating risks during periods of significant market downturns.

4.3 Evaluation of risk−adjusted performance

Table 5 reports the out−of−sample perform− ance of the different strategies, both excluding and including transaction costs, utilizing the Sharpe ratio (Sharpe, 1966), the modified VaR Sharpe (Favre and Galeano, 2002), and the Sortino ratio (Sortino and Price, 1994). In both instances, the REG strategy markedly outper− forms across all risk−adjusted return metrics, highlightingits superior capacity to maximize returns when adjusted for risk exposure, as well as its proficient management of downside risks. Although the maximum drawdown of the REG strategy is not the smallest (−0.166 without transaction costs and −0.179 with), it remains within an acceptable range, signifying robust risk management protocols. The improvement in the risk−adjusted performance not only pre− sentsthe model's predictive accuracy but also enhances the risk−return dynamics of the tradi− tional asset allocation strategy.

Table 5

Out−of−sample performance for the different portfolio strategies. The S&P 500 index is used as a benchmark. 15 basis points per transaction is imposed as costs in Panel B. The sample window is M = 180 months.

	Ann. Sharpe	MVaR Sharpe	Sortino
Panel A: Ex. transaction costs			
Benchmark	0.657	0.434	0.754
REG	0.735	0.554	0.761
EW−R	0.544	0.473	0.589
EW−S	0.428	0.348	0.472
MVSC	0.543	0.463	0.599
MINC	0.430	0.351	0.473
Panel B: Including transaction costs			
REG	0.589	0.461	0.614
EW−R	0.462	0.411	0.503
EW−S	0.423	0.345	0.467
MVSC	0.528	0.453	0.583
MINC	0.425	0.348	0.467

[5] Conclusion

This paper introducesa simple approach to portfolio optimization by integrating re-gression and machine learning techniques into the traditional Mean-Variance Optimization (MVO) framework. The empirical results dem-onstrate that this regression-based approach to portfolio management not only simplifies the optimization process but also enhances performance, even when accounting for the increased transaction costs associated with frequent rebalancing. The methodologies and techniques discussed here offer practical ave-nues for investors and financial analysts to en-hance their asset allocation strategies, making them more responsive to market dynamics and investor preferences.

Our study not only provides a fresh per-spective on portfolio optimization but also showcases the substantial potential of re-gression and machine learning models in en-hancing portfolio management. Future re-search could encompass a spectrum of in-strumental variable estimation techniques, ranging from linear predictability models to more complex approaches for nonlinear relationships. This would potentially bring new levels of adaptability and precision to portfolio optimization.

[6] Appendix A: Supplementary data

A.1 Portfolio rebalancing and terminal wealth, M = 120

In this section and the subsequent ones, we conduct a performance analysis of the portfolio selection strategies outlined in the main article, using a rolling window of M = 120 months. We observe that the reduced estimation win-dow introduces greater sampling variation in the estimates, which in turn, affects the per-formance metrics of the various portfolio se-lection strategies. However, the fundamental conclusions drawn in the main article remain substantially unchanged. Table A.1 provides a summary of the portfolio turnover analysis reached by each of the strategies. We present the results for M = 180 as well for comparison.

As illustrated in Figure A.1, Panel A, the REG portfolio, without considering transaction costs, consistently outperforms other com-parative portfolios in terms of cumulative re-turns, except during the initial few months. When transaction costs are considered, as shown in Figure A.1, Panel B, the REG portfo-lio's cumulative returns remain among the highest across most months. During the COVID-19 recession, the REG strategy demon-strated the smallest drawdowns compared to

Table A.1
Descriptive statistics of portfolio turnover.

	N	Mean	Std	Min	Max
M = 120 months					
REG	155	0.638	0.571	0.011	1.482
EW−R	155	0.395	0.335	0.006	1.200
EW−S	155	0.021	0.009	0.000	0.075
MVSC	155	0.085	0.071	0.018	0.798
MINC	155	0.022	0.011	0.007	0.099
M = 180 months					
REG	95	0.510	0.526	0.010	1.320
EW−R	95	0.329	0.290	0.007	0.866
EW−S	95	0.021	0.010	0.000	0.075
MVSC	95	0.072	0.078	0.026	0.751
MINC	95	0.022	0.011	0.008	0.083

Figure A.1

Cumulative return of the different strategies based on an initial investment of $1, using a rolling window of size M = 120 monthly observations. The shaded period corresponds to the crisis period. Panel A excluding transaction costs, Panel B including transaction costs of 15 basis points per transaction.

other strategies, consistent with results from the 180–month analysis, reaffirming its superior risk management capabilities.

A.2 Evaluation of risk–adjusted performance, M = 120

Table A.2 reports out–of–sample performance by evaluating portfolio strategy returns using the same metrics as in the main article. Again, we include the corresponding results for M = 180 as well for comparison. It confirms that with a rolling window of 120 months, the risk–adjusted performance of the REG strategy not only continues to outperform other strategies but also becomes more pronounced. This evidence supports the conclusion that the higher transaction costs associated with REG's frequent rebalancing are effectively offset by its performance benefits.

Table A.2

Out-of-sample performance for the different portfolio strategies. The S&P 500 index is used as a benchmark. 15 basis points per transaction is imposed as costs in Panel B.

	Ann. Sharpe	MVaR Sharpe	Sortino
Panel A: Ex. transaction costs			
M = 120 months			
Benchmark	0.616	0.402	0.751
REG	0.652	0.446	0.747
EW-R	0.372	0.363	0.389
EW-S	0.308	0.240	0.349
MVSC	0.377	0.294	0.414
MINC	0.322	0.255	0.362
M = 180 months			
Benchmark	0.657	0.434	0.434
REG	0.735	0.554	0.554
EW-R	0.544	0.473	0.473
EW-S	0.428	0.348	0.348
MVSC	0.543	0.463	0.463
MINC	0.430	0.351	0.351
Panel B: Including transaction costs			
M = 120 months			
REG	0.454	0.323	0.526
EW-R	0.269	0.268	0.285
EW-S	0.303	0.236	0.344
MVSC	0.359	0.281	0.395
MINC	0.317	0.251	0.356
M = 180 months			
REG	0.589	0.461	0.614
EW-R	0.462	0.411	0.503
EW-S	0.423	0.345	0.467
MVSC	0.528	0.453	0.583
MINC	0.425	0.348	0.467

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포트폴리오 최적화를 위한 회귀 접근법

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Abstract

본 연구는 회귀 및 머신 러닝 기법을 전통적인 Mean-Variance Optimization (MVO) 프레임워크에 통합하여 포트폴리오 최적화에 대한 편리한 접근방법을 제안한다. MVO와 Ordinary Least Squares (OLS) 회귀 간의 연결을 확립하고, 회귀 분석을 통해 자산 배분을 재정의한다. 본 연구의 방법론은 회귀 분석을 위한 목표 수익률을 생성하고, 머신 러닝 알고리즘을 적용하여 포트폴리오 관리를 강화한다. 이 접근법은 포트폴리오 최적화를 재해석할 뿐만 아니라 동적 금융 시장에서 예측 모델의 잠재력을 시사한다.

주제어 : 평균-분산 최적화, 회귀, 예측 가능성, 기계 학습.
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