JKSCI

한국컴퓨터정보학회논문지
Journal of The Korea Society of Computer and Information
Vol. 30 No. 1, pp. 241-249, January 2025
https://doi.org/10.9708/jksci.2025.30.01.241

Reliable Growth Estimation Using Theil-Sen: A Comparative Study in AI-Driven ALEKS Program

Hyunsuk Han*, WANG XINYI**

*Assistant Professor, Dept. of Psychology, Jeonbuk National University, Jeonju, Korea
**Graduate Student, Dept. of Psychology, Jeonbuk National University, Jeonju, Korea

[Abstract]

The recent acceleration in the expansion of Artificial Intelligence (AI) powered learning platforms has enabled the collection of large volumes of learning data, thereby overcoming the limitations of traditional data collection methods. This study focuses on analyzing progress data from students using ALEKS, an AI-based online platform, and compares the ordinary least squares method with the non-parametric Theil-Sen method for estimating progress slopes. The findings indicate that while both models produce similar slope estimates, the Theil-Sen model is more reliable, especially when dealing with extreme values, which are often present among low achievers. These results highlight the necessity of considering alternative analytical methods when parametric estimation methods are inappropriate, to obtain a reliable estimates. Furthermore, this study underscores the need for ongoing research to explore more precise methods for analyzing AI-driven data, particularly in light of the limitations of parametric assumptions inherent in traditional estimation models.

▸ **Key words**: Progress monitoring, Theil-Sen, OLS, Estimation, ALEKS

[요 약]

최근 인공지능(AI) 기반 학습 플랫폼의 확산이 가속화되면서, 대량의 데이터 수집이 가능해져 기존의 데이터 수집 방법의 한계를 극복할 수 있게 되었다. 본 연구에서는 AI 기반 온라인 플랫폼인 ALEKS를 활용하여 수집된 학생들의 학습 진전도 데이터를 분석하여 성과 추정치를 산출하기 위해 일반적인 최소 제곱법과 비모수적 테일-센 방법을 비교하였다. 연구 결과, 두 모델 모두 유사한 회귀계수 추정치를 산출하지만, 특히 저성취 학생들에게 자주 나타나는 극단값에 대해서는 테일-센 방법이 더욱 신뢰할 만한 것으로 나타났다. 이러한 결과는 모수적 추정 방법이 부적절한 경우, 적합한 추정치 산출을 위하여 비모수적 분석 방법을 고려해야 함을 시사한다. 또한, 본 연구는 기존의 추정 모델에 내재된 모수적 가정의 한계를 고려하여, AI 기반 데이터 분석에 있어 더욱 정밀한 방법을 탐구하기 위한 후속 연구의 필요성을 강조한다.

▸ **주제어**: 진전도 점검, 테일-센, OLS, 추정, ALEKS

• First Author: Hyunsuk Han, Corresponding Author: Hyunsuk Han
*Hyunsuk Han (hysh@jbnu.ac.kr), Dept. of Psychology, Jeonbuk National University
**WANG XINYI (mollymolly12345678@gmail.com), Dept. of Psychology, Jeonbuk National University
• Received: 2024. 11. 01, Revised: 2024. 12. 23, Accepted: 2024. 12. 27.

I. Introduction

Progress monitoring (PM) is widely used to evaluate students' performance and verify the effectiveness of interventions [1]. PM results play a crucial role in decision-making and improving learning outcomes by providing individualized feedback for each student [2]. The role of monitoring students' progress has become increasingly important due to the decline in academic performance among students who experienced the shift to online public education during the COVID-19 pandemic [3,4]. In general, progress monitoring requires collecting data at regular intervals to track student performance over time [5]. However, data collection is not only hindered by challenges in ensuring consistent measurements but is also subject to external factors like student participation and health conditions. Additionally, teacher competence and subjective judgment in the evaluation process can impede the collection of accurate and consistent data. The expansion of online learning, accelerated by COVID-19, has made it possible to collect vast amounts of learning data using digital learning platforms, addressing the limitations of traditional data collection methods. The use of an online learning platform offers significant advantages in measurement [6]. For instance, it eliminates restrictions on the location, time, and frequency of measurements. Therefore, the widespread use of online learning tools has made it possible to collect and analyze data needed to provide individualized feedback to students who require special intervention.

Progress monitoring within a response to intervention model is typically achieved using curriculum based measures (CBMs). Until recently, classic test theory-based reliability indices were primarily used in the literature to sufficiently support using CBM for repeatedly measuring progress over time [7,8]. Growth over time is typically analyzed through slope and calculated using ordinary least squares (OLS) [7,9,10]. In PM, OLS regression models the relationship between PM scores and time. The strength of the time-score relationship is assessed by computing the sum of squared residuals from the regression model. OLS models have been widely employed as quantitative analytical models across various behavioral sciences [11].

OLS remains the preferred statistical analysis to reliably estimate growth in time series data but other analytical techniques have also been recommended [12,13]. They investigate the robustness of these methods under various conditions, including their ability to meet parametric assumptions (normality, equal variance) and accurately represent the growth of low-performing students. The findings suggest that Theil-Sen is more robust than OLS, particularly when dealing with outliers. The robustness of the Theil-Sen method to outliers has led to its widespread application in fields such as financial valuation and environmental science [18]. However, its application in other disciplines, such as learning and education data analysis, has not been fully explored. With the COVID-19, the growth of online learning platforms presents data quality challenges, making Theil-Sen's approach a more accurate and reliable alternative to traditional OLS in learning analytics. The purpose of this study is to compare Theil-Sen and OLS estimation to evaluate their effectiveness in estimating progress monitoring slopes based on empirical data from AI-driven ALEKS.

Theil-Sen slope is comparable to the parametric regression slope, which can be interpreted as growth over time [14]. Theil-Sen provides a robust alternative to parametric methods, especially when dealing with data that violate parametric assumptions, such as outliers or non-constant variance along the regression line. [15]. The Theil-Sen estimator is a robust statistical method used to estimate the slope of a linear relationship between two variables. It is particularly useful when the data contains outliers or is not normally distributed, as it is less sensitive to outliers than OLS regression. The Theil-Sen estimator calculates the median of the slopes of all possible pairs of points in the dataset using the following formula.

$$N = \frac{n \times (n-1)}{2}$$

If there are n data points (i.e., the number of weeks by a student was recorded), the total number of slopes(N) is equal to the number of pairs of two data points chosen from n, and for each pair the slope is calculated. The final estimate of the slope is the median of these slopes. The utility of the Theil-Sen model in reliably estimating growth has not been applied to progress monitoring studies. The emerging literature also suggest the need to continue to explore analytical methods that will more precisely estimate results and include improved reporting practices to include estimates of variability (i.e., trendedness indices, standard errors, and confidence intervals) [12,16]. An index of score variation indicates the stability of the results over time and is recommended as part of the analysis [16]. Further, in decision making circumstances that may result in changes in service delivery or eligibility, research urges reporting measurement error [12,16]. Reporting reliable and consistent growth requires just a few indices which are part of the routine output from the OLS regression model. A trendedness index tells us the percent of the scores that follow the slope line closely. In the OLS model trendedness is reported through an R^2 value. The index is important because it might have a slope of 0.32, but an R^2 value of 0.10 suggesting that approximately ten percent in this example of the scores are actually contributing to growth. Another index is the standard error of the slope indicating score variation [17]. More score variation results in lower R^2 values. The standard error of the slope is calculated to construct confidence intervals (CIs) that indicate the range within which the true slope likely falls. Best practice suggests that slope be reported with a trendedness index, Standard error of the slope and CIs provide to more precisely estimate performance [12,16]. This study aims to analyze student progress using data collected from ALEKS, an AI-based online platform. This study will compare the commonly used OLS method with the non-parametric Theil-Sen model for estimating progress monitoring slopes.

II. Method

This study evaluates the performance of ordinary least squares (OLS) and Theil-Sen models in estimating individual student growth rates based on progress monitoring data from ALEKS. This study aims to evaluate the reliability and technical adequacy of each approach to ensure accurate decision-making.

1. Participants

This study analyzed progress data from the pre-calculus course offered by ALEKS by McGraw-Hill. Data from 79 college freshmen was analyzed over a 15-week period. ALEKS does not provide any personal information about learners; it only provides weekly learning progress. All participants were freshmen enrolled in the College of Life Sciences. These students were taking a college mathematics course, and their performance on ALEKS was reflected in their grades. Data from students who did not complete the course or failed to log into ALEKS after the first two weeks was excluded.

2. Instruments

ALEKS is an AI-powered platform that assesses individual student knowledge and tailors learning paths accordingly. Students can access ALEKS anytime on computers or tablets to continue their learning. In this course, ALEKS was used as an assignment tool. Instructors created course plans by selecting various problem types (topics) related to the subject. Students' performance on these topics was factored into their grades.

ALEKS offers personalized learning paths to students. While instructors may register 100 topics for a course, not all students need to study all 100 topics. ALEKS uses an initial knowledge check to identify mastered topics, excluding them from the student's learning path. For example, a student who masters 30 of 100 topics will focus on the remaining 70.

The course structure in ALEKS is divided into two formats: Module-based and Self-paced. In this study, the Module-based format was applied. In the Module-based format, instructors assign topics to various modules and set deadlines for each module. In the Self-paced format, students are free to learn the topics throughout the semester at their own pace. Table 1 presents an overview of the ALEKS platform's key features used in this study.

Table 1. An Overview of the ALEKS Platform's Key Features

Feature	Description
Initial Knowledge Check	Assessing topics students already know.
Personalized Pathway	Establishing learning pathways based on the initial knowledge check.
Performance Tracking	Recording topics completed to measure progress.
Class Completion Knowledge Check	Identifying the topics that have been truly mastered among the learned content.

In this study, the progress of students is measured by the number of topics they have completed. Students engage with topics by logging into ALEKS and solving each topic's problems 3-5 times. ALEKS determines topic mastery based on student accuracy. Student progress in this study was assessed by the number of topics completed. The analysis was based on 14 weeks of data, excluding the final exam period.

3. Procedures

Weekly scores for 79 participants were recorded, but analysis focused on 78 due to one student's incomplete data. Linear regression analysis was conducted on each student's PM data, utilizing both OLS and Theil-Sen. The OLS output included slope, R^2, standard error of the slope, normality, heteroscedasticity, and autocorrelation tests. To complement the OLS analysis, Theil-Sen slope and Tau-U, a trendedness index, were calculated using the zyp package in R. OLS evaluated the linearity of the data, and the Tail-Sen method calculated the median slope, showing strength in nonlinear data. All analyses were performed using R 4.4.1.

The overall process is as follows. (see Figure 1) It starts with data collection on the AI-driven ALEKS platform, and then goes through cleaning (excluding data from students who have not completed the course) and model selection. Finally, the OLS and Theil-Sen methods are used to quantitatively estimate students' learning progress, and the results (slope and trend indicators) are compared and analyzed. The process focuses on the data of low-achieving students to further verify the robustness of the Theil-Sen method in handling extreme values.

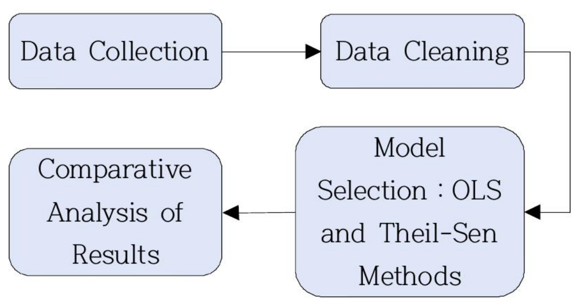

Fig. 1. Research Procedure

III. Results

This study evaluated the performance of OLS and Theil-Sen in estimating slopes. We compared the accuracy of their estimates and assessed their reliability using trendedness and standard error. Additionally, we examined the data from low-achieving students to determine if it violated the parametric assumptions of normality and homoscedasticity.

1. Parametric Assumptions of Normality and Equal Variance

Several tests were used to determine if parametric assumptions were met among this group of students. These results are presented in Table 2. The Shapiro-Wilk test was used to evaluate the normality of the slope coefficients. Shapiro-Wilk test statistics (W) ranged from 0.87 to 0.99, with corresponding p-values indicating varying degrees of normality. Specifically, all the participants had W statistics that met the normality assumption. The assumption of homoscedasticity was tested using a non-constant error variance test. At an alpha level of .05, the test statistics fell between 0.00 to 10.13, indicating that 15.4% of the sample had error terms that were not constant. The Durbin-Watson test was used to examine whether the residual errors over time were correlated. At a significance level of $p \le .05$, approximately 60.3% of the cases showed positively autocorrelated residual error terms.

Table 2. Tests of Normality and Equal Variance Assumption

Parametric Analysis				
Test	Assumption met	n	Percent	Range
Shapiro-Wilk	Yes	78	100	0.87 to 0.99
	No	0	0	
Non-constant Error Variance	Yes	66	84.6	0.00 to 10.13
	No	12	15.4	
Durbin Watson	Yes	31	39.7	0.47 to 2.70
	No	47	60.3	

Note. Shapiro-Wilk test of normality, W statistics $p \le .05$ significance level indicate error terms were not normally distributed; Non-constant error variance test $p \le .05$ significance level; *observed one participant violated tests for both equal variance and autocorrelation; Durbin-Watson test of residuals $p \le .05$ significance level

2. Comparison of Theil-Sen and OLS Slopes Coefficients

The Theil-Sen and OLS slope coefficients were compared to evaluate differences in estimation among participants. Table 3 presents descriptive statistics for both methods. The mean OLS slope was slightly higher (5.25, SD = 1.32) than the mean Theil-Sen slope (4.97, SD = 1.49). While median values (OLS: 5.16, range = 8.32; Theil-Sen: 4.86, range = 9.65) and overall slope magnitudes were similar, indicating general agreement between the methods, individual slopes varied. Notably, for one participant, the Theil-Sen slope was meaningful while the OLS slope was not significant (see Figure 2). The results show that the outliers have a significant effect on the slope estimation of OLS. The OLS slope is 5.29, 95% confidence interval is wider, [-0.86, 11.43]. This interval contains negative values, indicating that the estimate is unreliable. By comparison, Theil-Sen has a median slope of 2.67. This estimate is lower and more stable. The OLS regression line often deviates from the true trend and shows a large margin of error, especially in the presence of outliers. By contrast, the Theil-Sen regression line follows the trend of the data. Table 4 shows that outliers make the OLS slope about 98.13% higher than the Theil-Sen slope. This highlights the robustness of Theil-Sen's approach, especially when analyzing underachieving students.

The study's charts show that Theil-Sen's method better handles progress data for underachieving students. It can more effectively capture actual trend changes. This is of great significance for educational intervention and personalized learning paths. Accurate slope estimates help to more reliably reflect students' real progress.

Table 3. Descriptive Results for Slopes of ALEXS Participants

Slopes	Range	Mean	Median	SD
OLS Estimates	8.32	5.25	5.16	1.32
Theil-Sen Estimates	9.65	4.97	4.86	1.49

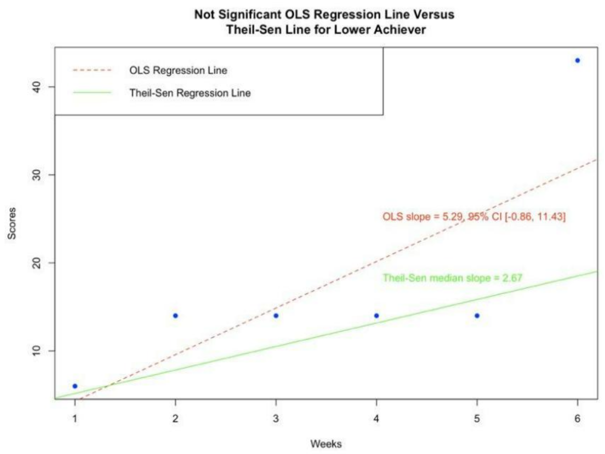

Fig. 2. Example of a Regression Results for a Low Achievement Students

Table 4. Example of the Outlier Impact Change Rate for a Low Achievement Students

Metric	OLS	Theil-Sen
Slope Estimates	5.29	2.67
Outlier Impact Change Rate(%)	98.13%	

Note. $Rate of change = \frac{OLS\ Slope - Theil-Sen\ Slope}{Theil-Sen\ Slope}$

3. Slope Diagnostics

Table 5 presents the slope stability diagnostics, including both parametric and non-parametric trendedness measures. The stability of the slopes was assessed using the R^2 index (parametric) and Kendall's Tau-U (non-parametric). The R^2 index is interpreted as the percentage of the data that contributes to linear improvement, while Kendall's Tau-U indicates the percentage of data showing any improvement (Parker et al., 2011). While these two indices are not directly comparable, they both offer valuable insights into different aspects of the trend.

In an OLS analysis, the R^2 index reflects the consistency of score contributions to growth over time. Kendall's Tau-U, a non-parametric measure analogous to R^2, assesses the overall trend without assuming a specific data distribution. In this study, Kendall's Tau-U values for the 78 participants ranged from 0.07 to 1.00, with a value of 0.91 indicating that 91% of scores demonstrated improvement over time. The average R^2 of the OLS method is 0.87. This means that linear time trends account for about 87% of the variance. However, some individual data points may not respond well to the model. These trendedness indices offer complementary perspectives on the data: R^2 emphasizes linear improvement, while Tau-U captures the overall direction of change. Table 5 illustrates how these measures can be used together to assess the stability of growth patterns in the data. This study introduces a more robust progress monitoring method, especially for underachieving students. The results show that Theil-Sen method is more reliable than traditional OLS method. It can better handle outliers and improve the accuracy of estimates.

When combined with AI-driven learning analytics tools, the approach supports personalized educational interventions. In the future, it can be applied to other disciplines and educational levels to confirm its wide practicality. These findings drive the development of educational data analysis. They also provide a solid foundation for better decision-making and smarter allocation of learning resources.

Table 5. Diagnostic Results for Slope Coefficients

Trendedness	Range	Mean	Median	SD
R^2 (OLS)	0.86	0.87	0.89	0.13
Tau (Theil-Sen)	0.93	0.91	0.92	0.07

Note. R^2= Coefficient of determination; Tau= non-paramentric index of percent of data pairs contributing to improvement

IV. Discussion and Conclusions

This study analyzes progress data collected through ALEKS, an AI-based learning platform. The goal is to compare the performance of the ordinary least squares (OLS) and nonparametric Theil-Sen methods. It also aims to assess the reliability of both methods. The study collected 15-week study data from 78 first-year students in the College of

Life Sciences. Both the OLS and Theil-Sen methods are used to estimate student progress. The findings of the current study indicate that while both models produce similar slope estimates, they differ in their technical adequacy. This study compared diagnostic indices that can be obtained using either model and are crucial for practitioners analyzing PM data to accurately summarize performance. While most participants showed similar slope estimates from both models, one case revealed the limitations of OLS. Despite low student achievement, the Theil-Sen estimator demonstrated significant progress, suggesting that OLS may not always be sufficient. A large percentage of participant results failed to meet one or more parametric assumptions, highlighting the need for caution when using OLS. Our results also suggest that Theil-Sen may be more sensitive to the data of low-achieving students. This study compared additional diagnostic indices more recently identified in the literature as being essential for using PM results for high stakes decisions [12,16]. Both OLS and Theil-Sen models produced similar indices (R^2 and Tau-U, respectively) useful for assessing the stability of the results. These data suggest that the type of progress monitoring data seen for low-performing students may not be optimally suited for OLS regression analysis alone.

The rapid expansion of online education, accelerated by the pandemic and the advent of the Fourth Industrial Revolution, has brought about revolutionary changes across education, healthcare, industry and etc. In particular, the decrease in the school-age population, the revision of the high school curriculum, and the diversification of college admissions processes have exacerbated issues such as the decline in basic academic skills and the widening achievement gap among university students. Consequently, in higher education, AI-driven adaptive systems are being used to create online learning environments that enhance foundational academic skills and address issues like learning disparities and dropout rates. AI-based adaptive systems like ALEKS analyze learners' knowledge and competence, delivering personalized learning that addresses their strengths and weaknesses efficiently. So, individual students, with distinct levels and objectives, can achieve optimal outcomes by progressing at their own pace within a personalized learning environment. From this perspective, AI-based adaptive systems effectively support learners who may have been left behind in traditional classrooms, thereby reducing the achievement gap among learners. These results suggested that, when monitoring progress for low achievers, the Theil-Sen method is recommended as an alternative when OLS assumptions are violated.

This study sought to address issues related to analyzing and summarizing PM results. PM results are typically analyzed using OLS, and in this study we compared slopes obtained from OLS with slopes from a non-parametric model, Theil-Sen. We also compared other indices routinely required of other high stakes assessments and recommended in recent literature to reliably and precisely summarize PM results. We found that slopes and other indices were similar between the models but differences were noted that call into question the technical adequacy of OLS for low performing students. Based on the results of this study (e.g., violations of parametric assumptions and statistically significance of Theil-Sen results), more research and debate is necessary if PM continues to be recommended for decision making within an RtI framework.

This study introduces a more reliable method for monitoring students' learning progress, aiming to improve the accuracy of learning assessments for low-achieving students, provide these students with personalized learning path solutions, and seek to optimize the allocation of educational resources, ultimately hoping to support and improve their academic progress. But this study has limitations that may affect both internal and external validity. The participants were undergraduate students in general mathematics courses, not a nationally representative sample. While various background variables (e.g., gender, socio-economic status, academic motivation etc.) likely influenced the results, ALEKS only provided learning data, limiting the analysis of relationships with other variables. Additionally, the definition of "low achievers" was constrained by the available data, limiting the generalizability of the findings to populations beyond "low achievers." These findings also limit the generalizability of the results to a wider population. To solve this problem, future research should use large-scale datasets to validate the robustness of Theil-Sen's approach. These data sets should include thousands or even tens of thousands of students from different regions and countries. Analyzing data from different educational stages and disciplines can also be helpful. This study compares the OLS method with the Theil-Sen method using data from the ALEKS learning platform. The results showed that the Theil-Sen method was more effective in analyzing data from underachieving students. However, the study did not explore data from other platforms such as Khan Academy, Coursera, or edX. To confirm the wider applicability and reliability of the method, future research should also test it on these platforms.

ACKNOWLEDGEMENT

“This paper was supported by research funds for newly appointed professors of Jeonbuk National University in 2023”

REFERENCES

[1] M. McInerney and A. Ellridge, "*Using a response to intervention framework to improve student learning: A pocket guide for state and district leaders*," American Institutes for Research, pp.1-18, 2013.

[2] T. D. Tolar, A. E. Barth, J. M. Fletcher, D. J. Francis, and S. Vaughn, "Predicting reading outcomes with progress monitoring slopes among middle grade students," Learning and Individual Differences, Vol. 30, pp. 46-57, February 2014. https://doi.org/10.1016/j.lindif.2013.11.001

[3] S. Hwang, S. Han, Y. Cho, H. Jeong, and J. Lee, “A systematic review on online education in mathematics education: Focused on before and after COVID-19,” Communications of Mathematical Education, Vol. 38, No. 2, pp. 93-120, January 2024. https://doi.org/10.7468/jksmee.2024.38.2.93

[4] Y. Y. Oh, "Problems and directions of mathematics education in the post-COVID-19 era," Journal of Learner-Centered Curriculum and Instruction, Vol. 23, No. 16, pp. 773-787, August 2023. https://doi.org/10.22251/jlcci.2023.23.16.773

[5] S. A. A. January, E. R. Van Norman, T. J. Christ, S. P. Ardoin, T. L. Eckert, and M. J. White, "Evaluation of schedule frequency and density when monitoring progress with curriculum-based measurement," School Psychology, Vol. 34, No. 1, pp. 119-127, January 2019. https://doi.org/10.1037/spq0000274.

[6] A. Alturkistani, A. Majeed, J. Car, D. Brindley, G. Wells, and E. Meinert, “Data Collection Approaches to Enable Evaluation of a Massive Open Online Course About Data Science for Continuing Education in Health Care: Case Study,” MIR Medical Education, Vol. 5, No. 1, April 2019, https://doi.org/10.2196/10982

[7] A. Foegen, C. Jiban, and S. Deno, “Progress monitoring measures in mathematics: A review of the literature,” The Journal of Special Education, Vol. 41, No. 2, pp. 121-139, August 2007. https://doi.org/10.1177/00224669070410020101

[8] M. M. Wayman, T. Wallace, H. I. Wiley, R. Ticha, and C. A. Espin, "Literature synthesis on curriculum-based measurement in reading," The Journal of Special Education, Vol. 41, No. 2, pp. 85-120, August 2007. https://doi.org/ 10.1177/00224669070410020401

[9] S. L. Deno, L. S. Fuchs, D. Marston, and J. Shin, “Using curriculum-based measurement to establish growth standards for students with learning disabilities,” School Psychology Review, Vol. 30, No. 4, pp. 507-524, December 2019. https://doi.org/10.1080/02796015.2001.12086131

[10] P. M. Nelson, E. R. Van Norman, D. A. Klingbeil, and D. C. Parker, "Progress monitoring with computer adaptive assessments: The impact of data collection schedule on growth estimates," Psychology in the Schools, Vol. 54, No. 5, pp. 463-471, March 2017. https://doi.org/10.1002/pits.22015

[11] N. R. Draper and H. Smith, "*Applied Regression Analysis*", John Wiley & Sons, 1998.

[12] R. I. Parker, K. J. Vannest, J. L. Davis, and S. B. Sauber, "Combining non-overlapping and trend for single-case research: Tau-U," Behavior Therapy, Vol. 42, No. 2, pp. 284-299, June 2011. https://doi.org/10.1016/j.beth.2010.08.006.

[13] S. H. Mercer, A. F. Lyons, L. E. Johnston, and C. L. Millhoff,

"Robust regression for slope estimation in curriculum-based measurement progress monitoring," Assessment for Effective Intervention, Vol. 40, No. 3, pp. 176-183, April 2014. https://doi.org/10.1177/1534508414555705

[14] M. Hollander and D. A. Wolfe, "*Nonparametric Statistical Methods*", John Wiley & Sons, 1999.

[15] R. Wilcox, F. Clark, "Robust Regression Estimators When There are Tied Values," Journal of Modern Applied Statistical Method, Vol.12, No.2, November 2013. https://doi.org/10.56801/10.56801/v12.i.662

[16] T. J. Christ and M. Coolong-Chaffin, "Interpretations of curriculum-based measurement outcomes: Standard error and confidence intervals," School Psychology Forum, Vol. 1, No. 2, pp. 75-86, May 2007. https://doi.org/10.1080/02796015.2006.12088006

[17] J. C. Nunnally, "*Psychometric theory*", McGraw-Hill Book Company, 1967.

[18] J. A. Ohlson and S. Kim, "Linear valuation without OLS: The Theil-Sen estimation approach," Review of Accounting Studies, vol. 20, pp. 395-435, 2015. https://doi.org/10.1007/s11142-014-9300-0

Authors

Hyunsuk Han received the Ph.D. degree in Research Methodology from the University of Florida in 2018. Dr. Han joined the faculty of the Department of Psychology at Jeonbuk National University, Jeonju, Korea, in 2023.

He is currently a Assistant Professor in the Department of Psychology at Jeonbuk National University. He is interested in latent variable modeling, psychometrics, and applied statistics in social science.

WANG XINYI received her bachelor's degree in psychology from Jeonbuk National University in 2024 and is currently a graduate student in psychology at Jeonbuk National University.

WANG XINYI enrolled in Jeonbuk National University in September 2024. She has a strong interest in psychological statistics and measurement.