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가상조작물-반구체물-추상기호의 순차적 교수가 지적장애 학생의 수학 학습에 미치는 효과*

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The Effects of Virtual-Representational-Abstract Instructional Sequence on Math Learning for a Student with Intellectual Disability*

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ABSTRACT

[Purpose] Mathematics proficiency holds significant importance in the academic development of children with intellectual disabilities, while also playing a crucial role in a high quality of life. However, many students with intellectual disabilities have difficulty understanding abstract mathematical concepts and skills. Due to the special and individualized learning needs of these children, interventions that are highly adaptive and customized are essential. Therefore, this study was conducted to prove the effectiveness of the Virtual-Representational-Abstract(VRA) instructional sequence in facilitating mathematics learning for one elementary school student with an intellectual disability. **[Method]** The experiment had a was multiple probe across behaviors design focused on three math learning areas based on the participant's individual needs: place value, addition, and subtraction. The teacher that provided the intervention was the special education teacher at the student's school. The teacher conducted baseline, intervention, maintenance sessions with the student for 10 weeks, on a one-on-one basis, in a special class. As the independent variable, the VRA instructional sequence was conducted during three phases while using the app "Number Pieces", while drawing and while not using any tools. The researcher performed visual analysis for each math learning area, and calculated Tau-U values and PND. **[Results]** As a result of the study, it was found that intervention improved accuracy in all behaviors. The effect size of intervention and maintenance was very large at 1.00 for all behaviors. The researchers found a functional relation between the intervention package and the student's math accuracy. **[Conclusion]** The Virtual- Representational-Abstract instructional sequence had a significant positive effect on math learning for a student with intellectual disability

Key Words : Intellectual Disability, Mathematics, Virtual Manipulatives, Explicit Instruction, Instructional Sequence

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Ⅰ. Introduction

1. Significance of the Research

Acquiring math concepts and skills is a fundamental aspect of academic growth and significantly impacts living quality for individuals with intellectual disabilities, molding their capacity to engage in social interactions and lead self-sufficient lives. There has been a growing recognition of the unique mathematical learning challenges encountered by children with intellectual disabilities. These challenges are particularly pronounced when considering the diverse range of characteristics and abilities among individuals with intellectual disabilities. An in-depth comprehension of the intricacies involved in fostering mathematical skills in these individuals is necessary.

Students with intellectual disabilities often face substantial hurdles in reaching their full potential in mathematics education due to cognitive limitations, attention deficits, and low self-esteem. One of the key aspects that many students face is the difficulty in grasping abstract mathematical concepts and skills. These abstract ideas can pose significant challenges and may require specialized approaches to make them more accessible. Furthermore, organizing and sequencing mathematical steps can be particularly challenging for these students, which can hinder their ability to solve mathematical problems effectively. Students from initially disadvantaged backgrounds are at a higher risk of lagging behind their peers, subsequently impacting their progress in advanced mathematics as they continue their education, and this achievement gap may further widen over time (Jordan & Levine, 2009). This underscores the importance of early intervention and ongoing support to ensure that these students have the opportunity to bridge the gap and succeed in their mathematical learning journey. Given these challenges, students with intellectual disabilities need additional time and repetition to achieve proficiency in mathematics. Specifically, it is advisable to provide them with well-structured, intensive learning sequences, and personalized mathematics instruction (Schnepel & Aunio, 2022).

The most frequently employed intervention for students with disabilities combines hands-on and manipulative-based instruction, particularly in elementary education (Peltier al., 2020). This approach is rooted because it recognizes the significance of touching, moving, and seeing in fostering mathematical thinking, starting from the early stages of conceptual development and extending to the most advanced stages of comprehension (Baccaglini-Frank & Maracci, 2015). One intervention that stands out prominently in this context is the Concrete-Representational-Abstract (Below referred to as CRA) instructional sequence, which has proven to be effective in facilitating diverse mathematical learning contents involving students who had severe disabilities (Stroizeret al., 2015; Kaya & Yildiz, 2023; Spooner al., 2019). The CRA instructional sequence serves as a scaffolding framework, guiding students towards the ultimate goal of cultivating math skills at the abstract level through a carefully designed progression of hands-on manipulation and visual aids. Also use Concrete-Semi concrete-Abstract (CSA) as a term similar to CRA. There are only differences in terminology, but it can be seen as virtually the same technique (Kim, Kang, Seo & Shin, 2019). In domestic studies, we found that there were more studies using CSA terminology than CRA (Baek & Kim, 2019; Flores, 2022; Jung, Kim & Yoon, 2020; Kim & Baek, 2020; Ko & Lim, 2023; Lee, 2022; Lee & Kim, 2021; Park & Kim, 2007; Yoo & Kim, 2021).

Moreover, the adoption of technology-enhanced learning tools and platforms is increasingly popular as a response to the diverse needs of students and issues of instructional engagement (Wright, Knight & Barton, 2020; Lambert & Schuck, 2021). This trend has led to innovative approaches to learning being favored, particularly in the context of interactions with the environment. An exemplary innovation in this regard is the use of Virtual Manipulation (Below referred to as VM) as a compelling alternative to traditional Concrete Manipulation (Below referred to as CM). It employs an app-based or online-based way to digitize the objects that are being manipulated (Bouck, Working & Bone, 2017). This digitization not only aligns with the contemporary technological landscape but also demonstrates its remarkable potential to enhance engagement and alleviate cognitive load in the learning process. Research has consistently recognized the effectiveness of VM as an intervention (Bouck al., 2014; Bassette al., 2019; Jimenez & Besaw, 2020).

The Virtual-Representational-Abstract (below refereed as VRA) instructional sequence was born as an extension of the CRA instructional sequence, which has been shown successful. The initial two experimental studies were carried out on the fraction learning of special education children in middle schools, examining the effectiveness of the VRA and VA instructional sequences, respectively (Bouck et al., 2017a; Bouck et al., 2017b). With the accumulation of more dependable evidence supporting the VRA instructional sequence, the positive impact on various math learning domains for the developmental disabilities, and learning disabilities population is demonstrated. This integration of adaptation that reflects the integration of traditional teaching methods with assistive technology has also been validated by many research (Bouck al., 2018; Park, Bouck & Smith, 2020; Park, Bouck & Fisher, 2021).

Guided by such global research background, Korean educators and researchers have also expended considerable effort to explore innovative mathematical interventions for children with moderate disorders (Kang, Choi & Kim, 2023). And a well-balanced application of supplementary strategies such as self-monitoring, Touchmath, manipulatives, prompts, and metacognitive strategies were observed. But there only exist one study focusing on VM within the Korean educational context. The earlier study showcased a functional correlation between VM and enhanced self-confidence in fifth graders during fraction learning (Heo, Suh & Moyer, 2004). This research has illuminated the positive influence of VM in facilitating Korean students' improved comprehension of mathematical concepts, fostering both mental and visual understanding. And in a recent study in Korea (Park, 2023), the performance of the VA intervention in enhancing the fraction problem-solving skills of three middle school learners experiencing developmental disabilities was demonstrated. Whereas the utilisation of app-assisted teaching is not new in Korea, there is a huge blank in the research related to VM. This exploration should go beyond surface-level considerations, aiming to provide a more nuanced understanding of the challenges and opportunities associated with VM in the Korean educational landscape.

There is however a clear gap in research on effective math interventions specifically targeting the arithmetic skills of children with moderate intellectual disabilities in elementary school. As indicated by a synthesis (Long, Bouck & Kelly, 2023), most studies on Virtual Manipulation-based (VM-based) math interventions, whether VM appears in stand-alone or as a part of one intervention for children with developmental impairments have concentrated on middle school students, whereas the primary focus of very few research has been on elementary education levels. Furthermore, It was noteworthy that one research team, headed by Bouck and his associates, conducted the majority of the VM-based researches (Bouck, Park & Stenzel, 2020; Bouck, Long & Jakubow, 2021; Bouck, Long & Jakubow, 2022). While their research team's study addressed as many variables as possible, including developmental disabilities, learning disability populations at all grade levels, and the various forms of intervention. This concentration of research efforts within a single team, while valuable in terms of depth, consistency and reliability of research, highlights the urgent need to expand the scope of research backgrounds and results in this crucial area of study. Diversifying research perspectives and approaches can yield a more comprehensive understanding of the potential benefits and challenges associated with VM-based interventions in mathematics education.

2. Research Questions

Given this context, the implementation of the current study holds substantial significance. It serves as a critical step towards assessing the efficacy of the VRA instructional sequence in enhancing the math concept and skills of elementary school children with intellectual disabilities. This study is important in the context of Korean education since there are not many studies specifically designed to address the special requirements of Korean students, especially for the complete VRA intervention as well as for the elementary school population and elementary school maths content. Additionally, there is still a need for validation on the common and unique features of implementation in the Korean context. This current study was therefore designed to expand on the promising earlier studies and add to the amount of knowledge previously available on VRA instructional sequence, specifically focusing on their effectiveness in facilitating the mastery of mathematical learning among elementary level learners with intellectual disabilities. This study attempted to explore these research questions:

Research Question 1: Is offering the VRA instructional sequence to an elementary school student with intellectual disability effective in improving the student acquire addition, subtraction, and place value?

Research Question 2: Does the VRA instructional sequences enable the student to maintain acquired learning of addition, subtraction, and place value without any guidance?

Research Question 3: What are the perceptions of this intervention from the experimenter's and student's perspectives respectively?

Ⅱ. Method

1. Research Participant

An elementary school student participated in the study. The participant was selected for the study according to the following criteria: (a) have a diagnosis of intellectual disability; (b) ability to follow model and verbal instruction; (c) ability to recognize and write two-digit numbers; (d) ability to manipulate objects. Students who did not meet these requirements were excluded from the experiment. The researcher initially assessed six students requiring math support, as suggested by the experimenter. Following screening criteria and discussions with the experimenter, one specific student was identified. Subsequently, the intervention details were shared with the student's mother, and consent for participation in the experiment was obtained.

Tom, a third-grade student, was a 9-year-old male from Korea. He studied in a general elementary school in Gyeonggi-do, South Korea, he was eligible for help in special education. Tom was identified as having a mild intellectual disability based on K-WISC-V (the Korean version of the fifth edition of the Wechsler Intelligence Scale for Children), a specific IQ score is 48 (VCI 65, VSI 59, FRI 69, WMI 50, GAI 57). He displayed relatively strong communication skills and demonstrated the ability to follow directions effectively. However, during studying, he encountered challenges related to attention, which manifested as avoidance behavior.

Related to math, Tom demonstrated a pronounced strength in addition, showcasing a notable proficiency compared to other mathematical areas. He had achieved a solid grasp of recognizing two-digit numbers and could mechanically count up to approximately 70. The experimenter pretested on his mathematical ability using the BASA-EN (Basic Academic Skills Assessment: Number Sense) tests and BASA-M (Basic Academic Skills Assessment: Math). The results of BASA-EN are presented in 〈Table 1〉, and the results of BASA-M are presented in 〈Table 2〉.

〈Table 1〉 Pre-assessment of number sense

BASA-EN							
Sub-score (maximum score)				T Score	Percentile (%)	Percentile Ranges	Age difference score
Number Identification	Quantity Discrimination	Missing Number	Estimation				
56(80)	2(40)	27(30)	3(30)	33	4	5	1.9

According to the test results of BASA-EN (Kim, 2011), Tom's performance in the four parts of number sense varied greatly. He had better performance in number identification and quantity discrimination than missing number and estimation.

〈Table 2〉 Pre-assessment of grade-level math

BASA-M				
Test Level	T Score	Percentile (%)	Percentile Ranges	Grade Difference Score
3 grade level test	33.24	1	5	2.4
2 grade level test	35.94	4	5	1.9
1 grade level	41.99	24	3	2.5

BASA-M(Kim, 2007) is a test tool for evaluating arithmetic ability formulated according to the corresponding standards of math textbooks for grades 1, 2, and 3 in Korea. The experimenter followed strict rules and instructed Tom to complete the third grade, second grade, and first grade versions of the test in that order. In addition during testing it was found that Tom lacked a clear understanding of the concept of subtraction and solved them as addition problems.

〈Table 3〉 BASA of percentile range reference

Percentile Ranges	Percentile	Range Description
1	□ 〉 95%	Very excellent math level.
2	85% 〈 □ 〈 95%	Good math level.
3	15% 〈 □ 〈 85%	This is a normal level of performance.
4	5%〈 □ 〈 15%	Please provide guidance to improve basic math skills.
5	□ 〈 5%	Comprehensive and continuous mathematics instruction is needed.

These two tests provide a range of five levels of mathematical ability that match the results of the percentile (see Table 3). Corresponding to the standards could identify the need for additional mathematical support in many aspects of number sense. It also could be inferred that Tom's current math level aligned with that of a first-grader and corresponded to the gap between him and maths standards of second and third grade.

2. Experimental Design

A multiple probe across behaviors design which had been used in the similar type of studies (Bassette & Bouck, 2023; Bouck et al., 2019) was employed to demonstrate the effectiveness of VRA instructional sequences on the math learning of an elementary student with intellectual disability. This design was selected due to the homogeneity of the three math behaviors serving as dependent variables, while maintaining their inherent independence. Collecting baseline probe data in lieu of continuous data considered the potential resistance responses of target student that may arise from the excessive examination. Also it was grounded on the assumption that the data will remain unaffected by external factors. According to the prior assessment, the difficult areas of the student were place values, two-digit addition with regrouping and subtraction without regrouping.

3. Measurement

The percentage of correct answers to the matching math problems during independent practice was the dependent variable. Each probe for addition with regrouping and subtraction without regrouping contained 5 questions, and the probe for place value contained ten questions. In order to maintain the objectivity of the test questions, a online random number generator (https://www.calculator.net/random-number-generator.html) was employed to determine each probe for every behavior in the phases of baseline, intervention, and maintenance.

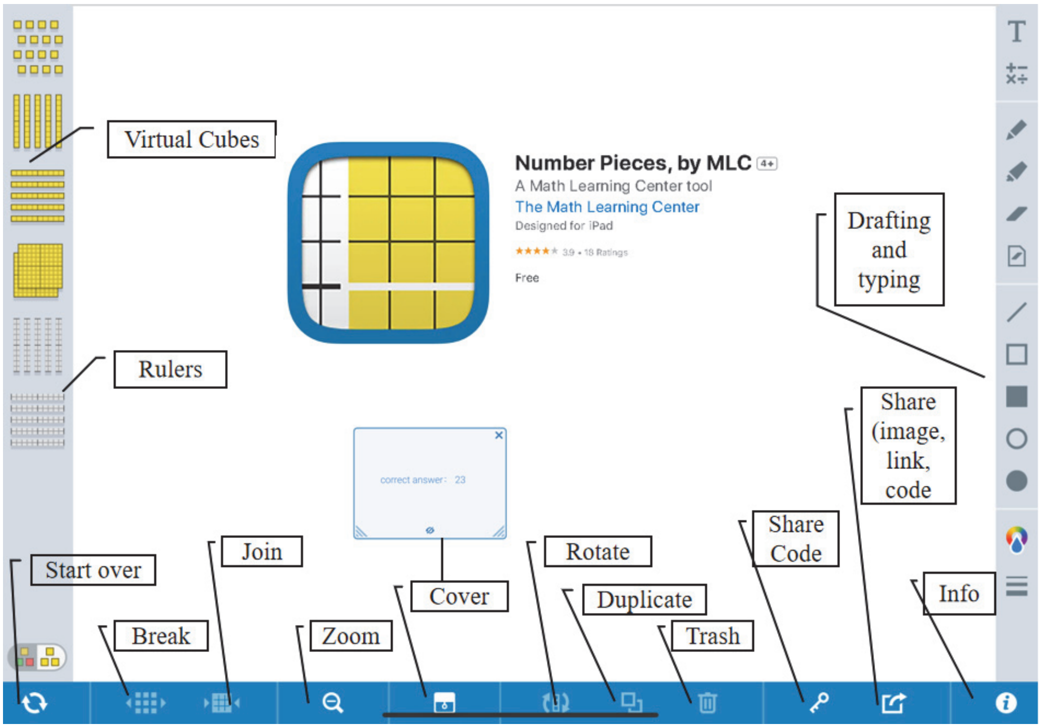
〈Table 4〉 Description of intervention goals

	Learning content	Types of questions	Examples
Tom	Place Value	Three-digit (Excluding 0)	111-999
	Addition	Two-digit + two-digit (with regrouping) (including Sum≤100, also Sum≥100)	57+78; 29+49;
	Subtraction	One-digit - one-digit (without regrouping) Two-digit - one-digit (without regrouping)	8 - 4; 16 - 1;

4. Setting and Materials

This research was carried out at a general elementary school in northwestern Korea, where about 3% students were receiving special education. All phases of the research, including baseline, intervention, and maintenance phases, occurred in the regular classroom setting to ensure student's comfort and familiarity. Each session was conducted at a same desk arrangement, with the teacher and student facing each other. The desk was equipped with exercise paper, pencils, and pads, devoid of any additional distractions. Video recorder was placed at right angles to the direction the students and teacher were facing to fully record student and teacher behaviour for later data collation and assessment.

A 10.9-inch tablet and an electronic stylus similar in size to a fountain pen were used for the baseline and virtual phase of intervention. The representational and abstract phase for the three math behaviors were taught and assessed on A4-sized paper. Within each session, there were two examples of modeling and guided instruction each, accompanied by one-page independent practice, thus amounting to a total of five sheets. The representational phase of the place value intervention was facilitated through the provision of printed images on paper, illustrating specific combinations of objects.



〈Figure 1〉 Screen information of “Number Pieces”

The virtual manipulation app selected for this study is the “Number Pieces” used in a previous study (Bouck, Long & Jakubow, 2021) to solve problems during virtual phase. Above all the app had the same characteristics of instruction and use as other virtual manipulation apps in terms of full functionality, good visualisation and ease of operation. The important fact or influencing the selection of this app is its cost-free accessibility and the added convenience of not requiring registration and login. Although this is an English-language program, the control panel is in no language, and all the functional graphics become easily comprehensible once they have been explained. Both the researchers and the experimenters agreed that its operation was more uncomplicated and direct compared to other similar applications available in the application market. In the baseline session, this study used the “Notability” app to present the questions, which includes three simple functions of free writing, eraser and page zoom, which were only needed during the test.

5. Researcher- experimenter Collaboration

All intervention details were thoroughly discussed and mutually agreed upon by the researcher and the experimenter. The special education teacher, who also served as the experimenter (the second author), took on the responsibility of implementing the intervention phase, as well as conducting the testing during the baseline, intervention, and maintenance phases for the participating student. She had seven years of expertise in special education, was pursuing a doctorate in the same field, and had spent the last two years overseeing students' educational needs. The researcher referenced prior researches and took responsibility for designing the experiment. This included establishing the experimental framework, defining goals aligning with the participant's capability, determining mobile app and supplementary materials, outlining specific intervention procedures, and formulating the assessment of dependent variables.

6. Procedure

This research implemented the way of one-to-one, with each session of no more than 30 minutes. Sessions conducted 2-3 times every week for about 10 weeks. In each session of each phase, the teacher used explicit instruction, including three parts: modeling, guided instruction and independent practice (see Figure 2). According to prior researches (Bouck et al., 2018; Bassette & Bouck, 2023; Bouck et al., 2019), each type of manipulation conducted in a minimum of 3 intervention sessions but not limited to three (i.e., three in the virtual phase, three in the representational phase, and three in the abstract phase).

1) Teacher Training

The researcher engaged in a collaborative effort with the teacher to exemplify and model the instructional process and experimental design. After the teacher had familiarized the whole framework, in which the teacher modeled lessons with a non-participating student on three different days. The model lesson contained three target skills and three teaching models in the most efficient way possible: (a) place value-virtual, (b) addition-representational, and (c) subtraction-abstract.

The researcher watched the recorded lesson and scored the lesson based on a pre-prepared treatment fidelity checklist. Feedback and discussions regarding potential enhancements were then undertaken collaboratively between the researcher and the teacher. In instances where treatment fidelity fell below the 80% threshold, the teacher would select another student for a second round of simulation lessons to reassess. This iterative process ensured strict adherence to the defined teaching methods and promoted continuous improvement in the fidelity of the experimental frameworks developed.

2) Baseline

The student required at least three stable trends of zero acceleration or deceleration before entering the intervention phase (Ledford & Gast, 2014). The baseline for the first skill (place value) consisted of 10 questions each session. During baseline sessions assessing the second and third skill (addition and subtraction), the student solved five questions respectively. The questions of each session were shown on the iPad, and student had access to the device to acquaint himself with tablet usage before the official virtual phase intervention. But the teacher was not allowed to provide any objects, prompts or feedback to the student.

3) Intervention

(1) Virtual Phase

In this phase, The teacher helped the student to use the “Number Pieces” App to perform VM through iPad and electronic pens. Prior to beginning formal instruction, the experimenter must verify that the student could proficiently use the app, eliminating the risk of incomplete problem-solving due to difficulties with app navigation. Following the teacher’s explanation of the function’s usage, assessment would be conducted on both the function’s execution and its correlation with numerical operations. Instruction would proceed until the student could independently perform all action. The intervention began with the least challenging skill, place value, transitioned to subsequent learning while becoming familiar with the operation of the application. The think-aloud strategy was implemented during modeling to simulate thinking process by externalize problem-solving steps. Modeling involved how to use digital colored blocks to represent the corresponding number, how to merge and split blocks of 10 units, and how to use drag to express the calculation process of addition and subtraction. After modeling two problems, the student was asked to solve three problems under the guidance of physical and verbal cues that were provided according to his needs. Finally, student independently completed the five-question assessment using the VM.

(2) Representational Phase

The representational phase taught the student how to solve targeted problems by drawing(or pictures instead). Since place value involved larger two-digit numbers, problems demonstrated by showing printed pictures on which the teacher and student drew their solutions. For addition and subtraction, problems were solved by drawing lines to represent ones. It was the same procedure as in the virtual phase, the teacher presented two examples, and after tutoring student to answer three questions, they progressed to the independent practice to assess students' accuracy by drawing to complete five questions.

(3) Abstract Phase

In each session of the abstract phase, problems were solved abstractly without using any object manipulation or drawing. The use of numerical strategies were incorporated to assist student in the process of abstraction, with particular attention to the selective use of fingers for counting. The teacher encouraged student to use numerical strategies start from modeling with two questions and guidance instruction with three questions to reduce the difficulty of transitioning from representational to abstract phase. Finally, student completed 5 questions independently for each skill.

(4) Maintenance

Two weeks after the last abstract phase of each skill, a total of two maintenance sessions were conducted. The maintenance probes were consistent with baseline (ten questions for place value, five questions for addition and subtraction). Any prompts, objects or feedback were not allowed for the student.

7. Inter-observer Agreement, Intervention Fidelity and Social Validity

The researcher collected inter-observer agreement (IOA) data on accuracy by observing 33% of the sessions in each of the three skill phases. Agreement or disagreement (Whether the researcher agreed that the experimenter was correct in judging the students' answers) was established by reviewing the responses recorded on the students' exercise sheets. The formula for calculating the inter-observer agreement (IOA) percentage was as follows: the percentage was calculated by multiplying the result by 100 after dividing the total agreements by the sum of the total agreements and disputes (Ledford & Gast, 2014). Every phase saw a 100% accuracy IOA from the student.

Treatment fidelity was assessed to ensure that the student was provided with appropriate material at each stage (i.e., virtual, representational, and abstract stages), that each math behavior was guided according to the appropriate step-by-step process, and that the intervention followed a clearly instructional process. A third individual other than the researcher and experimenter assessed the reliability based on a detailed checklist. This calculation was derived by recording 33% of sessions for all phases of the three skills, achieved by dividing the number of agreements by the sum of agreements and disagreements and then multiplying by 100% (Billingsley, Whiten & Munson, 1980). The researcher received 96.7% treatment fidelity for all skills across all phases.

Following the intervention, the researcher conducted social validity survey with both the participant and the experimenter to engage in discussions regarding their perceptions of the intervention. To indicate the degree of satisfaction, each question was answered using a Likert scale (the scale ranges from 1, indicating strong disagreement, to 5, signifying strong agreement). The researcher inquired with the student about their opinions on each phase (i.e., virtual, representational, and abstract phase) and which phase they preferred. The teacher was also subjected to interviews regarding various aspects, such as the reasonableness and importance of the intervention goals for the student, the effectiveness of the VRA instructional sequence in enhancing the targeted math behaviors, and the overall feasibility of the intervention process for the student.

8. Data Analysis

The researcher employed visual analysis to calculate the levels, trends, and effect sizes of each skill at every phase. Regarding levels, data stability was defined as 80% of the data falling within 25% of the median value (Gast & Spriggs, 2014). The split-middle method (White & Haring, 1980) was utilized to determine data trends. For each behavior in each phase, this method determined the middle point, calculated the mid-rate and mid-date, drew a line between the data to assess whether the line exhibited acceleration, deceleration, or zero-celeration (Gast & Spriggs, 2014). The researcher used Tau-U values and percent of non-overlapping data (PND) values to determine effect sizes. In which an online calculator: http://www.singlecaseresearch.org/calculators/tau-u was applied for the calculation of Tau-U values (Parker et al., 2011). A Tau-U value less than or equal to 0.65 indicated a small effect, between 0.66 and 0.92 suggested a moderate effect, and value greater than 0.92 was indicative of a large effect (Parker & Vannest, 2009). The calculation of PND involved dividing the total number of intervention data points surpassing the baseline value by the overall number of intervention sessions available.

Ⅲ. Results

1. Place Value

The initial focus for this student's math behavior centered on three-digit place value. During the baseline assessment, Tom struggled to complete any problems accurately, and the baseline stability level showed zero-celeration. Notably, a marked jump was observed immediately after the first intervention session, with he successfully solving all problems. Throughout the subsequent virtual, representational, and abstraction phases, Tom consistently met the established standard in a minimum of three intervention sessions, showcasing an impressive range of 100%-100%. A comparative analysis between the baseline and the overall intervention effects revealed a substantial effect size, as indicated by the

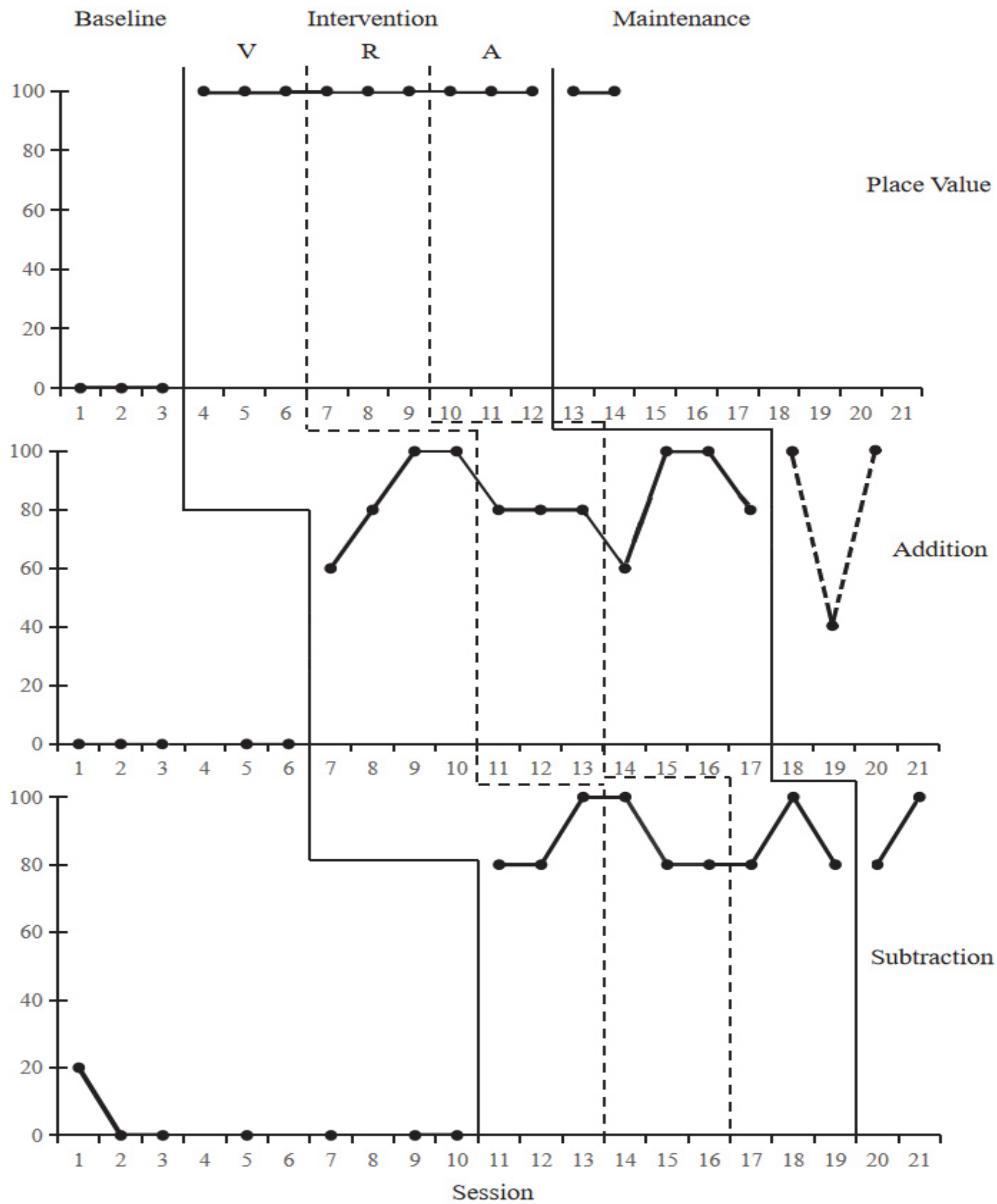
Tau-U value of 1.0. PND values(100%) between baseline and maintenance also demonstrated the effectiveness. He also maintained 100% accuracy during the maintenance phase two weeks later.

2. Addition

The second math behavior targeted for the student involved solving two-digit addition problems with regrouping. Tom failed to complete any problems accurately and maintained zero-celeration during baseline phase. However, A gradual progression became evident in the virtual phase (range= 60%-100%). He immediately reached the 80% mastery standard in the second intervention session. Following this achievement, the student's accuracy stabilized at 100% for the next two sessions, indicating a sustained grasp of the targeted math behavior by using VM. And he achieved 80% mastery criterion at all of representational phase. During the abstract phase, Tom met criteria across four sessions (one session repeated, range= 60%-100%). So Tau-U score of addition between baseline and overall intervention was 1.0. And between baseline and maintenance phase, Tau-U score(1.0) and PND value(100%) were indicating a very large effect size.

3. Subtraction

The student completed subtraction problems without regrouping as his third maths skill. His highest accuracy of baseline was 20%. Subsequently, across the virtual, representational, and abstract phases, Tom consistently achieved accuracy rates ranging from 80% to 100% in a minimum of three sessions. Notably, the Tau-U effect size for subtraction within the VRA instructional sequence was calculated as 1.0 and did not require any session repetitions. There was also a high accuracy for maintenance phase range from 80% to 100% (Tau-U=1.0, PND=100%).



〈Figure 2〉 Tom's math level

4. Social Validity

Following the survey results of the experimenter, there was a notable emphasis on the positive impacts of VM in expediting knowledge acquisition. She underscored its advantages in cultivating students' interest and confidence in learning, particularly when addressing basic level learning objectives. However, the experimenter identified a need for additional practice to enhance maintenance during subsequent abstract phase. While acknowledging the high adaptability of the VRA instructional sequence, the experimenter expressed reservations about the practical implementation of teaching more complex concepts such as multiplication, division, and fractions without a guided demonstration. The experimenter maintained a neutral attitude on this concern and ultimately assigned a satisfaction rating of 96% to this intervention.

Tom expressed 100% satisfaction with the entire learning process, highlighting his particular enjoyment of utilizing VM. He is satisfied with the achieved learning outcomes and has notably gained confidence in tackling addition and subtraction calculations. Tom is eagerly anticipating the continuation of learning other mathematical content through the VRA instructional sequence.Following the survey results of the experimenter, there was a notable emphasis on the positive impacts of VM in expediting knowledge acquisition. She underscored its advantages in cultivating students' interest and confidence in learning, particularly when addressing basic level learning objectives. However, the experimenter identified a need for additional practice to enhance maintenance during subsequent abstract phase. While acknowledging the high adaptability of the VRA instructional sequence, the experimenter expressed reservations about the practical implementation of teaching more complex concepts such as multiplication, division, and fractions without a guided demonstration. The experimenter maintained a neutral attitude on this concern and ultimately assigned a satisfaction rating of 96% to this intervention.

Ⅳ. Discussion

This research employed a contemporary and newly devised mathematical intervention, integrating VM and delivered through explicit instruction based on the traditional CRA instructional sequence. The VRA instructional sequence flexibly integrated the diverse and unevenly developed math learning needs of the target students so that the student could cultivate multiple mathematical concepts and skills within an unified teaching and learning framework. This intervention facilitated the setting of three different mathematical goals within a student, each varying in difficulty but precisely aligned with the developmental level of the student. The results showed that the student successfully achieved and maintained his accuracy in three-digit place value, two-digit addition with regrouping, and subtraction without regrouping. This underscores the effectiveness of the VRA instructional sequence and establishes a clear functional relationship among the intervention on the student's enhancement across each math learning goals. The conclusions and discussion of the research results are as follows.

First of all, the VRA instructional sequences showed the following results in the intervention sessions of place value, addition, and subtraction. Once the intervention was initiated, the student presented immediate progress on all three learning goals as with the previous study's results (Park, Bouck & Smith, 2020).

In terms of 'place value', Notably, the mastery of three-digit place value stood out as particularly impressive, being attained and consistently maintained at 100%, indicating the student's efficient grasp of this concept through the intervention. The experimenter attributed this notable improvement to the efficacy of VM. Early successes acted as a catalyst, instilling confidence and motivation throughout the learning process, and were the driving force behind the successful performance of the subsequent representational and abstract phases.

In terms of 'addition', the addition of two-digit with regrouping required the most intervention sessions which was evidently of some difficulty for him. Nevertheless, the positive outcomes observed during the maintenance phase were encouraging and indicated progress in overcoming the initial difficulties. It is significant to take into account that there were actually two tests conducted on the second result from the maintenance phase for addition, with the second result considered the final one. The experimenter reported a lack of student attention during the first test, leading to a mere 40% accuracy. Upon analyzing the test's pattern of errors (such as writing 157 instead of the correct result: 156), it was concluded that there was no inherent issue with the problem-solving. This study believed that the test results in this case should have been honestly reflected and marked differently (dotted line) as a cautionary note to future researchers to be more careful in their judgement of the test results.

Also in terms of 'subtraction', In contrast to the other two goals that require advanced learning based on existing level, subtraction posed a unique challenge for this student. The initial challenge rooted vagueness surrounding the concept of subtraction before the intervention. Therefore, the student's first focus was on differentiating between the concepts of addition and subtraction and mastering subtraction problems without regrouping. Once he had clearly grasped the concept of subtraction through the visual and hands-on intervention, the minimal sessions of intervention were achieved. Sequential and explicit instruction based on virtual manipulatives, as mentioned in the previous study (Park, 2023), had a positive impact on the concept of discrimination. The abstract phase of subtraction brought attention to a concern when employing finger counting as a strategy for comprehension. When the difference of subtraction was greater than 11 (which means counting on more than two hands), students with special needs may encounter challenges attributable to memory overload. Additional memory strategies become essential to mitigate this difficulty, such as seeking assistance from a peer for memory and prompting, or recording on paper. During the maintenance phase of subtraction, positive results were obtained but there was an accelerating trend, suggested that more consistent outcomes could potentially be observed with more number of sessions.

Secondly, the VRA instructional sequences showed the following results in the maintenance sessions of place value, addition, and subtraction. The purpose of the research was to cultivate the students' eventual capacity to independently arrive at abstract problem-solving. Initial findings from the study indicated a positive trajectory that notable progress was observed in aligning the student's problem-solving skills with the desired level of independence and abstraction. In the case of independent practice of all behaviors in the abstract and maintenance phases without any prompts given to the student, he autonomously utilize representational approach to complete tests few times.

However, in contrast to the full steps outlined in the formal instruction during the representational phase, the student independently simplified the utilization only when necessary. This dependency also tended to decrease significantly as the instruction progressed during the abstract phase. However, in the maintenance phase two weeks later, this dependency was seen to increase again during problem solving. In the maintenance phase after two weeks, though, this dependency was seen to increase again during problem-solving. Both the researcher and the experimenter interpret this wave as a necessary and positive sign indicative of the ongoing transition to abstract problem-solving. It is believed that, with continued practice, the student's ability to solve problems abstractly will become more stable over time.

Thrid of all, this study verified the effectiveness of the VRA instructional sequences and at the same time became a special study different from the previous studies. The first thing to discuss about the research design method is that this study borrowed the research design of previous studies. Among single-subject intervention studies in Korea, there are studies that continuously collect baseline in the 3 session before the start of intervention for each behavior. However, since many overseas study with the VRA instructional sequences as an independent variable did not collect baseline continuously (Bouck, Long & Jakubow, 2021; Bouck, Shurr & Park, 2020; Bouck et al., 2018), a follow-up discussion is needed on which baseline collection method is appropriate.

Furthermore, the results of this study support and extend the effectiveness of the VRA instructional sequence for elementary school students with moderate intellectual disabilities. While early researches on the VRA instructional sequence demonstrated its ability to cope with multiple ages of students in terms of social adaptability (Bouck et al., 2018), they have mostly concentrated on middle school pupils who have developmental problems. This may be due to the higher cognitive demands of VM, whereas the current study clarifies the adaptability of this intervention in upper elementary education. This can be ascribed to two aspects of the experiment's design. Firstly, during the baseline stage, students were instructed to utilize a versatile notes app on a tablet to respond to questions, representing an initial attempt into VM. The second cornerstone involved presenting the student with relatively simple mathematical conceptual content as the first objective. Hence, it could be theorized that introducing the VRA sequence to younger learners is likely to succeed with an appropriate preparatory process.

This study marks a departure from previous research conducted by the same research team, aiming to replicate and expand the intervention of VRA instructional sequence with other objective perspectives. Especially an additional distinctive feature of this study is the collaborative effort with Korean special education teachers, to deepen comprehension of the applicability and impact of the VRA instructional sequence within the framework of special education in Korea. Particularly when dealing with students with diverse and uneven learning needs, researchers are advised to collaborate with teachers who possess a deeper understanding of their students. It is also crucial for school teachers to have a comprehensive grasp of the learning objectives in different compulsory schooling contexts to effectively intervene with students. This study offers a more reliable reference for the future development of VRA sequences within the Korean educational context.

The practical implications arising from this study carry significant weight, particularly evident in the following domains. Firstly, The intervention sessions involved in this study lasted a minimum of 10 minutes and a maximum of 30 minutes. The positive outcomes derived from this intervention intensity prove to be highly effective for both students and teachers. This advantage is particularly evident in the case of Korean special education teachers when considering the professional challenges faced by special education teachers, who often need to teach across multiple disciplines while contending with a lack of specialized competence in certain areas (Park, 2023).

Secondly, the VRA instructional sequence stands out as an exceptionally inclusive intervention capable of addressing all facets of basic math learning to offer students individualized and personalized learning experiences. Because of its adaptability, sequences like VA, VR, and VRA can be used to fit the specific needs of many students as well as the needs of the same student with varying degrees of learning contents. Instead of the traditional CRA(or CSA) instructional sequence, which requires consideration of the shame that physical manipulation imposes on students in the upper grades, VM makes it more accessible to students at least at the middle school level (Bouck et al., 2018).

Thirdly, Another noteworthy practical implication of this research lies in the utilization of the app "Number Pieces." Within the scope of the learning content addressed in this study, the Number Pieces stands out for its better affordability and user-friendly interface. However, for subsequent research involving algebra, fractions or others, the Brainingcamp series will become a pertinent consideration which has more comprehensive maths learning functions. In summary, the study proposed that "Number Pieces" offered a user-friendly chance for all learners and parents from Korea to initiate their exploration of VM in a more convenient and financially stress-free way.

While the present study contributes valuable insights, it is essential to acknowledge and consider several limitations. A prominent limitation is that the intervention was carefully customized for one participant, hence the generalization of the findings should be treated with caution. Another limitation is whether the intervention can be seamlessly adapted to conditions under other settings. Settings such as traditional classroom teaching, small group learning, and home-based education exhibit different group interactions and pedagogical dynamics that may affect the effectiveness of the intervention in ways that differ from the one-to-one teaching model. In summary, the results of the experiment were an intricate interplay between the various factors of the intervention. The joint effects of this intervention on the math learning of students with intellectual disabilities remain largely unexplored beyond the current participant. Future research should consider replicating and extending the VRA instructional sequence for enhancing math learning in elementary school-aged children with intellectual disabilities.

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〈국문 초록〉

가상조작물-반구체물-추상기호의 순차적 교수가 지적장애 학생의 수학 성취도에 미치는 효과

포 월 · 정 다 이 · 손 승 현

[목적] 수학 능력은 지적장애 학생의 학업 발달에 매우 중요하며, 삶의 질에 많은 영향을 미친다. 그러나 많은 지적장애 학생들이 추상적인 수학 개념과 기술을 이해하는 데 어려움을 가진다. 지적장애 학생들의 개별화된 교육적 요구에 부합하는 맞춤형 수학 중재가 필수적이다. 따라서 본 연구는 지적장애 초등학생 1명을 대상으로 가상조작물-반구체물-추상기호의 순차적 교수를 실시하고 수학 학습에 대한 중재 효과를 검증하고자 하였다. **[방법]** 실험설계는 학생의 개별적 요구에 따른 자릿값, 덧셈, 뺄셈에 대하여 행동간 중다간헐기초선설계를 사용하였다. 중재자는 학생이 소속된 학교의 특수교사로 특수학급 교실에서 학생과 일대일로 10주간 기초선-중재-유지를 실시하였다. 기초선은 행동별로 3~7회기, 중재는 9~11회기, 유지는 2~3회기 수집하였다. 독립 변수로 'Number Pieces' 애플리케이션을 활용하여 학생이 조작을 하다가 점차적으로 도구를 사용하지 않는 가상조작물-반구체물-추상기호의 3가지 단계에 대한 순차적 교수를 설계하여 진행하였다. 수학 학습에 대하여 시각적 분석을 실시하였으며, 중재와 유지의 효과 크기를 알아보기 위하여 Tau-U와 PND 값을 산출하였다. **[결과]** 연구 결과, 가상조작물-반구체물-추상기호의 순차적 교수가 자릿값, 덧셈, 뺄셈에 대한 정확도를 모두 향상시키는 것으로 나타났다. 중재와 유지의 효과 크기가 세 목표행동 모두 1.00으로 매우 크게 나타났다. 가상조작물-반구체물-추상기호의 순차적 교수와 학생의 수학 문제 사이에 기능적 관계가 있음을 확인하였다. **[결론]** 가상조작물-반구체물-추상기호의 순차적 교수는 지적장애 학생의 수학 학습에 긍정적 효과가 있었다.

주제어 : 지적장애, 수학, 가상 조작, 명시적 교수법, 순차적 교수

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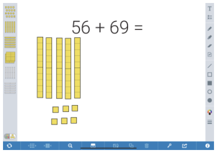
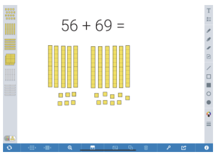
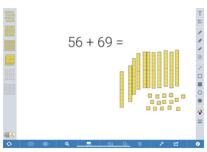
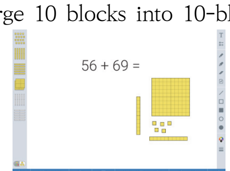
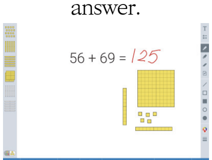
Appendix

Task Analysis of the VRA Phases Across Addition

Addition-Virtual (App-based)	Addition-Representational (drawing)	Addition-Abstract (Numerical strategy)
Read the equation.	Read the equation.	Read the equation.
Count and move the digital blocks for the first number. 	Draw lines and dots equal to the first number. 56 + 69 = 	Write down the numbers in a vertical format. 56 + 69 = 56 + 69
Count and move the digital blocks for the second number. 	Draw lines and dots equal to the second number. 56 + 69 = 	Start with the ones column. Add 6 and 9 to get 15. Write down 5 in the ones place and carry over the 1 to the tens place. 56 + 69 = 56 + 69 5
Drag down all digital blocks below the right side of the equals sign and count. 	Count all the lines and dots to the left of the equal sign. Sum can be written separately for memory. 56 + 69 = 	Move to the tens column. Add the carried-over 1, along with 5 and 6 to get 120. write down 2 in the tens place and carry over the 1 to the hundred place. 56 + 69 = 56 + 69 25
Use the "merge" function to merge 10-block into 100-block, and merge 10 blocks into 10-block. 	Count the final total, and write the answer. 56 + 69 = 125 	Move to the hundred column, write down the carried-over 1. 56 + 69 = 56 + 69 125
Count the final total, and write the answer. 		Write the answer. 56 + 69 = 125 56 + 69 125