

# Reexamination of Forward Premium Anomaly in Foreign Exchange Markets Allowing for Realized Volatility and Jump Process\*

실현적 변동성과 점프과정을 이용한 외환시장에서의  
forward premium anomaly 현상에 대한 재고찰

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한영욱

By using the daily Euro-Dollar and Yen-Dollar spot and overnight forward exchange rates, this paper reexamines the issue of the forward premium anomaly which has attracted widespread attention in international finance. In particular, this paper investigates a statistical evidence for the role of realized volatility and jump process in the tests of the anomaly, which have mostly ignored in the previous studies. For the purpose, this paper adjusts the usual regression model of testing the forward premium anomaly by allowing for realized volatility and jump process. After re-estimating the adjusted regression model, this paper finds that the estimated value of the forward premium coefficient in the Yen-Dollar currency market is positive while the coefficient in the Euro-Dollar currency market is still negative yet reduced significantly. Thus, this paper presents the possibility that the empirical phenomenon of the forward premium anomaly in foreign exchange markets may not be as robust as the previous studies have presented if the regression models used in those studies are specified more appropriately by allowing for the realized volatility and the jump process.

Key words: Bernoulli distribution, daily overnight forward exchange rates, daily spot exchange rates, Forward premium anomaly, high frequency exchange rates, jump process, realized volatility

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## I. Introduction

This paper is concerned with a new approach to examine the forward premium anomaly, which refers to the widely reported empirical result that most freely floating nominal spot exchange rate returns appear to be negatively correlated with the lagged forward premium. The anomaly implies that a non-negative forward premium would on average, result in an appreciating currency for the country with the higher rate of interest, which is the opposite to the theory of Uncovered Interest rate Parity (UIP). The forward premium anomaly has played the same degree of importance in international finance as the equity premium paradox in domestic asset markets, and has attracted corresponding widespread attention. Several explanations have been developed to explain the anomaly. Some of them include a time varying risk premium (Mark, 1985; Hodrick and Srivastava, 1986; Kaminski and Peruga, 1990), peso problems (Lewis, 1995; Evans and Lewis, 1995), irrational behavior of market participants and segmented markets (Frankel and Rose, 1995), habit persistence (Backus et al., 1993) and feedback from monetary policy (McCallum, 1994). However, as discussed by Mark and Wu (1998), Baillie and Bollerslev (2000), Maynard and Phillips (2001) and Maynard (2001), these explanations have as yet failed to reconcile the anomaly.

In particular, it should be noted the possibility that the forward premium anomaly may be due to the omitted variable in the regression model suggested by Fama (1984) for the test of the forward premium anomaly as pointed by Baillie and Bollerslev (1990) and Bekaert and Hodrick (1993). The usual regression model for the forward premium anomaly tests have omitted the variable of the conditional variance of spot exchange returns which is correlated with the forward premium. Most of the previous studies

which have tested the forward premium anomaly by using the usual regression model excluded the conditional variance term of spot exchange returns since it is empirically found to be very small and statistically insignificant so that it is difficult to measure (McCulloch, 1975; Cumby, 1988; Hodrick, 1989).

Also, there are recently strong theoretical reasons and overwhelming empirical evidence that the spot returns can be well described as an  $I(0)$  process and is close to being a martingale difference sequence but the forward premium has forms of long memory behavior, which implies a fractionally integrated  $I(d)$  ( $0 < d < 1$ ) process. Thus, more recent approach has emphasized the econometric issues involving unbalanced regression model. The different time series properties of the spot returns and the forward premium imply restrictions on the forward premium anomaly. This approach has been presented by Baillie and Bollerslev (2000), Maynard and Phillips (2001) and Liu and Maynard (2005)

Thus, this paper takes a different regression model for the test of the forward premium anomaly by allowing for the realized volatility and the jump process in order to reduce the problems in the usual regression model which have mostly ignored in the previous studies for the tests of the anomaly. This paper estimates the usual regression model for the test of the anomaly by using only the spot exchange returns and the forward premium of the daily Euro- Dollar and the Yen-Dollar exchange rates. Then, this paper adjusts for the jump process to reduce the long memory problem in the forward premium and the realized volatility to proxy the future spot return volatility in the usual regression model to resolve the problems in the regression model and re-estimates the adjusted regression model for the test of the forward premium anomaly.

This paper finds that when the jumps and the realized volatility are

allowed in the regression model, the forward premium coefficient in the JPY-Dollar currency markets is found to be positive implying that the anomaly may be disappeared and the coefficient in the Euro-Dollar currency markets is found to be negative but is reduced significantly suggesting that the anomaly still exists but the degree of the anomaly could be not as severe as before. Thus, this paper provides statistical evidence that the forward anomaly may not be as robust as the previous studies presented.

The plan of the paper is as follows: section 2 reviews the theoretical backgrounds relating the forward premium anomaly and discusses the problems in the usual regression model for the test of the forward premium anomaly: the presence of the long memory property in the forward premium and the omitted variable of the conditional variance of spot returns. Then, it provides the theoretical issues of the jump process and the realized volatility to reduce the problems in the regression model related to the tests of the forward premium anomaly. Section 3 provides the regression model without and with adjusting for the jump process and the realized volatility in the regression model for the test of the forward premium anomaly. And it gives the estimation results. Finally, section 4 concludes and discusses the implication of these findings in the context of the forward premium anomaly.

## II. Theoretical backgrounds: the forward premium anomaly, the realized volatility and the jump process

The forward premium anomaly refers to the widespread result that the returns on freely floating exchange rates are invariably negatively correlated with the lagged forward premium. In the following,  $S_t$  denotes the spot

exchange rate on day  $t$ , while  $F_{t, k}$  refers to the forward exchange rate on day  $t$  for delivery on day  $t+k$ . Corresponding logarithmic values are denoted by lower case variables letters,  $f_t$  and  $s_t$  for the spot and forward exchange rates. Further,  $i_{t,k}$  denotes the dollar return on a  $k$  period risk free dollar denominated bond on day  $t$ , whereas  $i_{t,k}^*$  is the foreign currency return on a risk free bond. The Uncovered Interest rate Parity (UIP) condition states that

$$E_t(\Delta S_{t+k}) = (f_{t,k} - s_t) = (i_{t,k} - i_{t,k}^*), \quad (1)$$

where  $E_t(\cdot)$  denotes an expectation conditioned on a sigma field of information available at time  $t$ .

Hence the forward premium,  $(f_{t,k} - s_t)$ , or equivalently the interest rate differential,  $(i_{t,k} - i_{t,k}^*)$ , are the expected rate of appreciation (depreciation) of the currency  $k$  business days in the future. In terms of the usual empirical work, the forward premium regression is often based upon data with a sampling frequency that exactly matches the maturity time of the forward contract, so that the test of the forward premium anomaly focuses on the following model,

$$\Delta S_{t+1} = \alpha + \beta_0(f_t - s_t) + u_{t+1}, \quad (2)$$

and tests that  $\alpha = 0$ ,  $\beta_0 = 1$  and  $u_{t+1}$  uncorrelated. But, the typical empirical finding of most previous studies is that the estimation of Eq.(2) provides a significantly negative of  $\beta_0$  (Froot and Frankel, 1989; Backus et al., 1993; McCallum, 1994). The forward premium anomaly refers to the widespread finding of a negative estimator of  $\beta_0$ , which implies that an

appreciating currency results for the country with the higher rate of interest.

Furthermore, it is helpful to have some insight into the foundations of the theory behind the regression in Eq. (2). The UIP hypothesis requires the joint assumptions of rational expectations, risk neutrality, free capital mobility and the absence of taxes on capital transfers. Hence expected real returns in the forward market must be zero, so that,  $E_t[(f_{t,k} - s_{t+1})/p_{t+1}] = 0$ , where  $p_t$  denotes the domestic logarithmic price level. By a Taylor series expansion of equation to second order terms,

$$E_t(s_{t+k}) - f_t = -0.5\text{Var}_t(\Delta s_{t+k}) + \text{Cov}_t(s_{t+k}, p_{t+k}), \quad (3)$$

where  $p_t$ , refers to the logarithmic price level

Note that even under rational expectations and risk neutrality, the right hand side of Eq. (3) contains the two conditional second moment terms. Generally, the discrete time, consumption based asset pricing model provides a risk adjusted equivalent to Eq. (3), which emphasizes real returns over the current and future consumption streams of the representative investor,  $E_t[(F_{t,k} - S_{t+k})/P_{t+k}] \cdot [U'(C_{t+k})/U'(C_t)] = 0$ , and is the intertemporal marginal rate of substitution. The analogue to Eq. (3) is,

$$E_t(s_{t+k}) - f_t = -0.5\text{Var}_t(\Delta s_{t+k}) + \text{Cov}_t(s_{t+k}, p_{t+k}) + \text{Cov}_t(s_{t+k}, q_{t+k}), \quad (4)$$

where  $q_{t+k}$  is the logarithm of the inter-temporal marginal rate of substitution, and  $\text{Cov}_t(s_{t+k}, q_{t+k})$ , is a time dependent risk premium ( $\rho_{t+k}$ ).

In an economy with generally low inflation and stable foreign exchange markets, it seems reasonable to omit the term  $\text{Cov}_t(s_{t+k}, p_{t+k})$  on the grounds that it is negligible. Hence, with data matching the sampling

interval of the maturity time of the forward contract, the more appropriate model for the empirical testing would be

$$\Delta S_{t+1} = \alpha + \beta_0(f_t - s_t) + \beta_1 \text{var}_t(\Delta S_{t+1}) + \rho_{t+1} + u_{t+1}. \quad (5)$$

It should be noted that an important issue concerns the term of  $\text{Var}_t(\Delta s_{t+1})$  in Eq. (5) even though the time varying risk premium term ( $\rho_{t+1}$ ) cannot be observable so that it could be excluded in the model<sup>1)</sup>. As Baillie and Bollerslev (1990) and Bekaert and Hodrick (1993) have pointed out, the forward premium anomaly may be due to the omitted variables, the volatility term of spot exchange in Eq.(5).

But, the volatility term represented by the conditional variance of foreign exchange rates is an important factor in foreign exchange markets since it can provide very useful information in the price discovery process of foreign exchange rates (Andersen, et al., 2001a). Thus, it is very important to understand how to measure the volatility of foreign exchange rates and how the dynamics evolves over time. In particular, it has been a key issue in the volatility research to find a reliable way to estimate the volatility of foreign exchange rates in the study of foreign exchange markets. Andersen et al. (2001a) have introduced a novel measurement of the volatility in foreign exchange rates, which is called as realized volatility.

The basic idea of the realized volatility is that a reliable measurement of

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1) Some previous studies including McCulloch (1975), Cumby (1988) and Hodrick (1989) have found that the volatility of spot exchange returns are very small and statistically insignificant so that it is difficult to measure exactly. And, even though several models like the optimizing models of consumption and the general equilibrium models have examined the foreign exchange risk premium, the models have generally been failed to work well. For the more survey related to the issues, see Engel (1996).

the exchange rate volatility can be proxy by the summation of squared returns over the relevant horizon by using high frequency data. Andersen et al. (2001b) have presented that the realized volatility constructed is not only model free but also measurement error free when the data frequency approaches infinity. Since the realized volatility approaches the underlying integrated volatility, the realized volatility can be considered observed rather than latent when the data frequency is high. And, they have provided that the realized volatility can provide an unbiased estimation of volatilities. Similarly, Barndroff-Nielsen and Shephard (2002) have proved that realized volatility is a consistent estimator of integrated variance under stable conditions.

Also, some papers like Baillie and Bollerslev (2000), Maynard and Phillips (2001) and Liu and Maynard (2005) have suggested that the different time series properties of the spot returns and the forward premium may induce the forward premium anomaly because of the unbalanced regression model in terms of the integration order. They have found that the spot returns can be well described as an  $I(0)$  process but the forward premium has forms of long memory behavior, which is a fractionally integrated  $I(d)$  ( $0 < d < 1$ ) process. In particular, the long memory property in the forward premium may be due to the presence of jumps or structural breaks<sup>2)</sup>. The recent work of Han (2010) has showed that the forward premium series in the Euro-Dollar and the Yen-Dollar foreign exchange markets contains very strong double long run persistence or long memory property in the first two

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2) Choi and Zivot (2007) and Sakoulis and Zivot (2001) has also showed that the structural breaks can generate spurious long memory property in the forward premium and that the long memory property can be substantially reduced when the structural breaks are appropriately accounted for. and, Choi et al. (2008) have suggested that both long memory and structural breaks should be considered together in conducting the tests of the forward premium anomaly.



moments and the jumps can generate spurious long memory property in the forward premium and that the long memory property can be substantially reduced when the jumps are appropriately accounted for.

As pointed by Han (2010), these jumps could be caused by the large changes in the foreign exchange rates, which are closely related to economic and/or financial events in foreign exchange markets. These events concerning expected future flows can result in large price changes well above normal and might be captured by jumps. These jumps might lead to the level and volatility outliers and result in the significant long memory property in the forward premium. Hence, it may be understandable that the jump process can proxy the unobservable risk premium in order to explain unexpected movements in the foreign exchange markets occurred by some economic and/or financial events and to resolve the statistical anomaly problem caused by the long memory property in the forward premium.

### III. Reexamination of the forward premium anomaly with allowing for the realized volatility and the jump process

In order to re-examine the forward premium anomaly, this paper focuses on a detailed analysis at the daily frequency level for the Euro-Dollar and the Yen-Dollar exchange rates which are the mostly traded currencies in foreign exchange markets. For the purpose, this paper uses the daily spot and overnight forward exchange rates and the 5-minute high frequency spot exchange rates of the Euro-Dollar and the Yen-Dollar during the periods of January 3, 2000 through December 31, 2007. The datasets are constructed after eliminating the weekends and the world wide holidays like Christmas

day (December, 25) and the New Year day (January, 1), in which the trading activities in foreign exchange markets are relatively low. These datasets are provided by Olsen & Associates of Zurich, in which Reuter FXFX quotes are recorded in time to the nearest second and the data sets consist of a bid and an ask price. In particular, the daily overnight forward exchange rate data with one-day maturity is used to avoid the problem of the data overlapping by matching the interval with the daily spot exchange rates. Also, the 5-minute high frequency spot exchange rates data is used in order to calculate the realized volatility in the Euro-Dollar and the Yen-Dollar foreign exchange markets following the suggestions of Andersen et al. (2001a)<sup>3)</sup>.

The spot ( $S_t$ ) and forward ( $F_t$ ) exchange rates for each daily interval is determined as the average between the bid and the ask quotes. Hence the daily spot return series for day  $t$  is the first differenced logarithm of the spot exchange rate and is approximately the continuously compounded rate of return, which is defined as  $R_t = 100 \times [\ln(S_t) - \ln(S_{t-1})]$ . The daily forward premium series for day  $t$  is defined as the difference between the daily overnight forward rates and the daily spot rates,  $fp_t = 100 \times [\ln(F_t) - \ln(S_t)]$ .

In order to understand some of the issues with the anomaly, it is convenient to consider some of the time series properties of the two different data. Table 1 summarizes the descriptive statistics for the daily spot returns and the forward premium series in the Euro-Dollar and the Yen-Dollar

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3) Andersen et al. (2001a) have suggested that use of equally spaced high frequency returns strikes a satisfactory balance between the accuracy of the continuous record asymptotic underlying the construction of realized volatility measures on the hand and the confounding influences from market microstructure frictions on the other hand. And, they have presented that one good choice of sampling frequency for the daily realized volatility is the 5-minute returns data which is high enough such that the daily realized volatility is largely free of measurement error.

exchange markets. For the spot returns, the returns of the Euro-Dollar is -0.018 on average providing the Dollar has been appreciated about 1.8% against the Euro during the sample period while the returns of the Yen-Dollar is 0.005 so that the Dollar has been depreciate about 0.5% against the Japanese Yen. But, the general movements of the spot returns in the two different currency markets appear to be quite similar. For the forward premium, the average values of the forward premium of the Euro-Dollar and the Yen-Dollar are -0.001 and -0.012 respectively presenting both the Euro and the Yen have been at a discount against the Dollar during the periods but the Yen has been discounted more than the Euro. Also, the higher variance of the forward premium in the Yen-Dollar shows that the forward premium appears to be more volatile in the Yen-Dollar currency markets compared to the Euro-Dollar markets.

The spot returns data and the forward premium data in the two different markets are found to have very different properties. There exists statistically significant excess skewness and excess kurtosis in the forward premium series<sup>4)</sup> indicating the evidence of non-normal distribution while the spot returns appear to follow the normal distribution with the skewness values close to zero and kurtosis value close to 3. And, the Ljung-Box test statistics,  $Q(20)$ , show evidence that the forward premium series markets contain significant autocorrelations, suggesting the possibility of the long memory property while the spot returns have relatively insignificant autocorrelations.

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4) According to Jarque and Bera (1987), the standard errors of the sample skewness and the sample kurtosis in their corresponding normal distributions are  $(6/T)^{1/2}$  and  $(24/T)^{1/2}$ .

〈Table 1〉 Descriptive Statistics

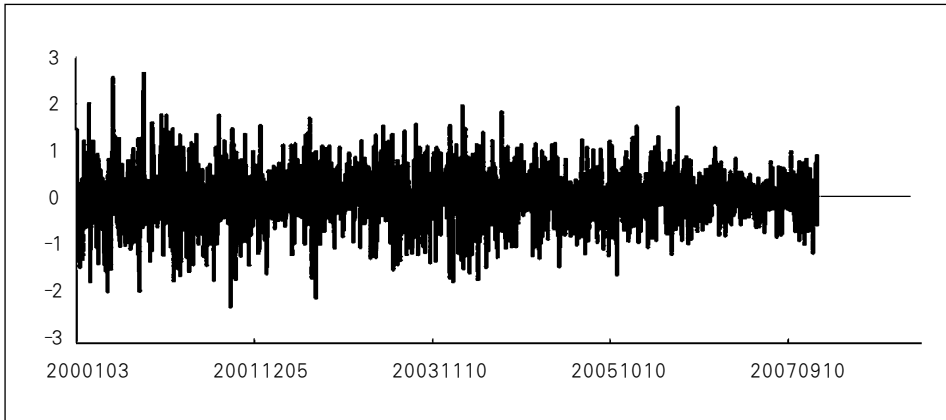
	Euro-Dollar exchange rates		Yen-Dollar exchange rates	
	Spot returns	Forward premium	Spot returns	Forward premium
Mean	-0.0176	-0.0014	0.0046	-0.0115
Median	-0.0152	-0.0013	0.0217	-0.0105
Max	2.2849	0.0231	2.4614	0.0028
Min	-2.6473	-0.0302	-2.8186	-0.0808
Variance	0.3394	0.00004	0.3352	0.0001
Skewness	-0.0661	-1.2902	-0.3269	-2.1010
Kurtosis	3.8655	5.1731	4.6103	5.9411
Q(20)	12.0451	22610.1366	7.5431	7074.6901

Note: Q(20) is the Ljung-Box statistics with 20 degrees of freedom based on the spot returns and forward premium.

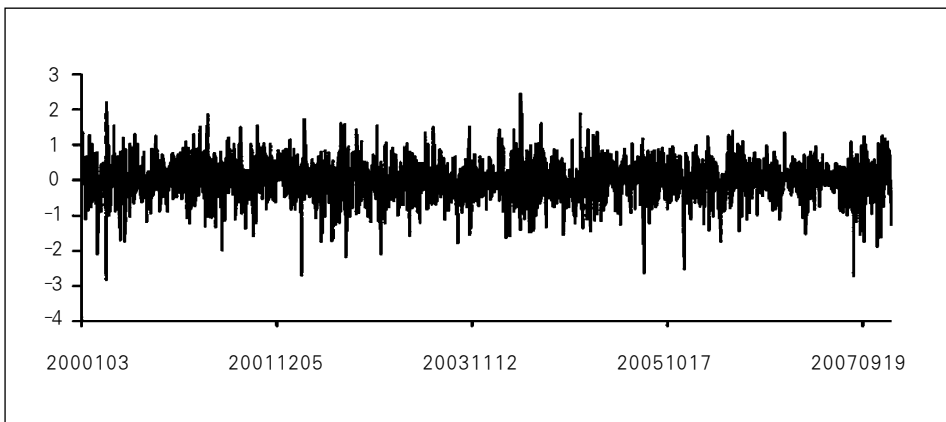
The basic data descriptions for the spot returns and the forward premium in the two currency markets can be confirmed by Figure 1 to Figure 4. From Figures 1 (a) and (b) which present the daily spot returns of the Euro-Dollar and the Yen-Dollar exchange rates, they are seen to exhibit the similar movement patterns with oscillating around zero. In contrast, the daily forward premium series of the Euro-Dollar and the Yen-Dollar exchange rates shown in Figures 2(a) and (b) exhibit very persistent movements with very distinct phases.

〈Figure 1〉 Daily Spot Returns

(a) Euro-Dollar Returns

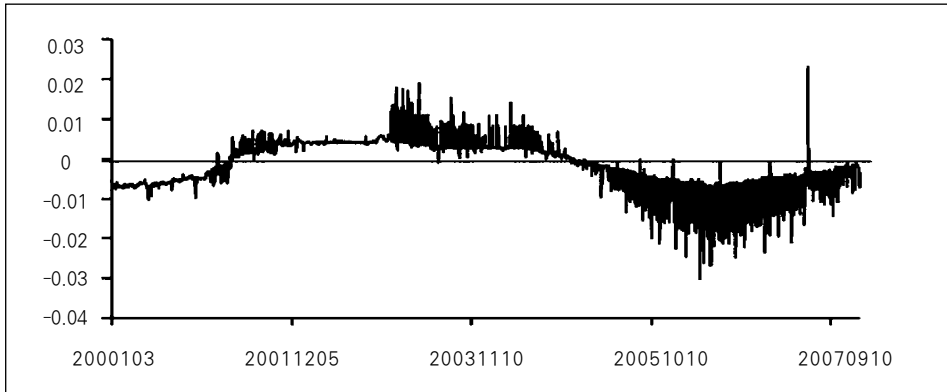


(b) JPY-Dollar Returns

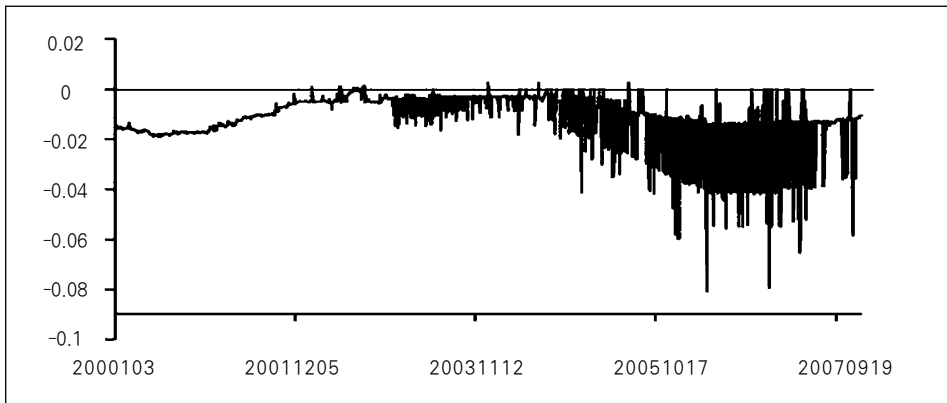


〈Figure 2〉 Daily Forward Premium

(a) Euro-Dollar forward premium



(b) JPY-Dollar forward premium

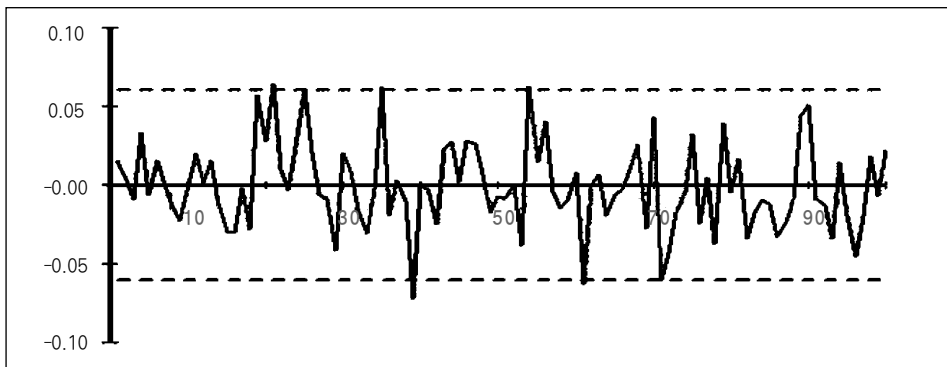


In order to investigate the inherent time series features, Figures 3 and 4 present the correlograms for the first 100 lags of the daily spot returns and the daily forward premium of the Euro- Dollar and the Yen-Dollar exchange rates. While the daily spot exchange returns are found to be close to being a martingale process, which are shown in Figures 3(a) and (b), the extreme

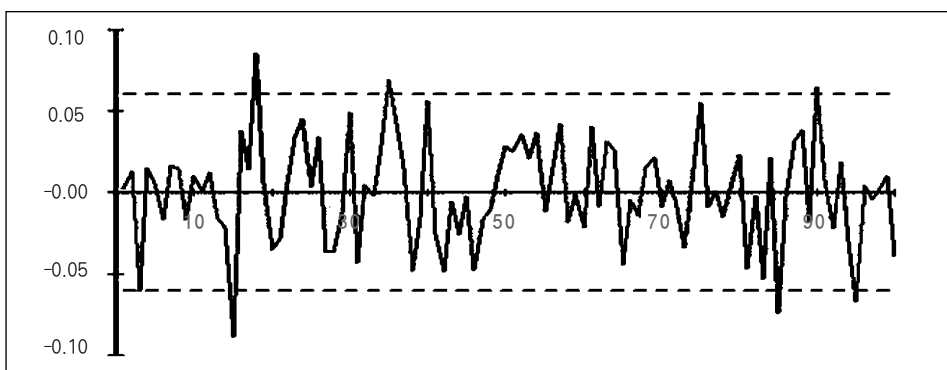
persistence of the autocorrelations in the forward premium renders presented in Figures 4(a) and (b) is quite different from that of the spot returns. The differences in the time series property between the spot returns and the forward premium are consistent with Baillie and Bollerslev (1994, 2000) and Maynard and Phillips (2001) who used long spans of daily and monthly exchange rates series.

〈Figure 3〉 Correlograms of Daily Spot Returns

(a) Euro-Dollar Spot Returns

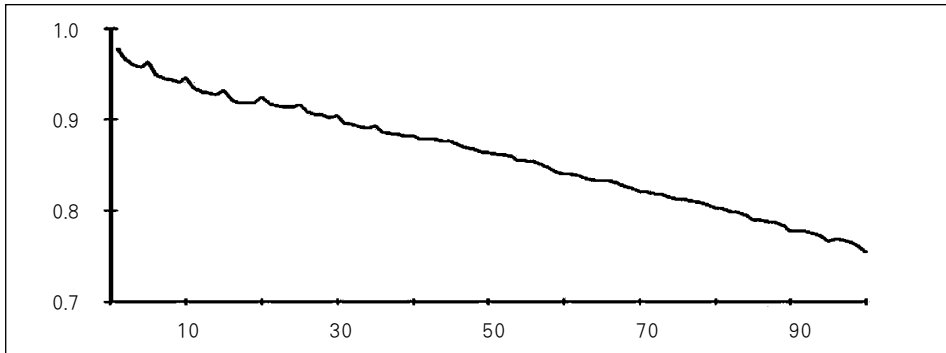


(b) Yen-Dollar Spot Returns

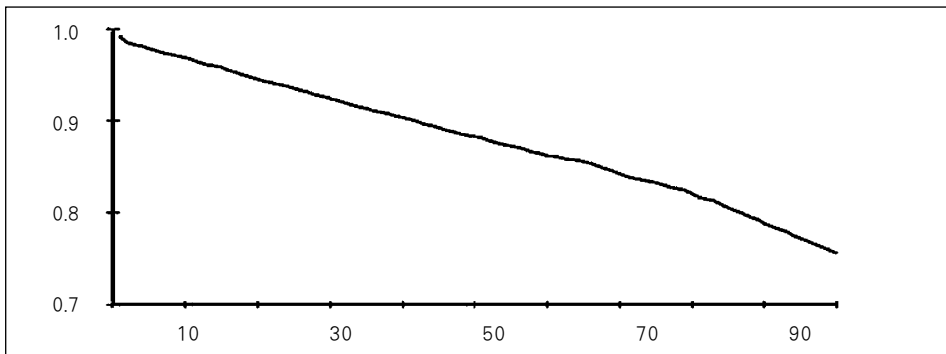


〈Figure 4〉 Correlograms of Daily Forward Premium

(a) Euro-Euro Forward Premium



(b) Yen- Dollar Forward Premium



The forward premium anomaly can be illustrated by using the usual regression model in Eq. (2). The regression model is estimated by using the QMLE method of Bollerslev and Wooldridge (1992) and the robust standard errors since the forward premium are not normally distributed and significantly serially correlated. As shown in Table 2, the estimated slope coefficients from the Eq. (2) are found to be  $-3.27$  and  $-0.06$  in the Euro-Dollar and the Yen-Dollar currency markets, which are statistically significant at the conventional significance level. Thus, the strong forward premium anomaly can be found in the two different markets. These



anomalous findings in the two currency markets are entirely consistent with the evidence reported in the existing literature for other currencies. Among them, Baillie and Bollerslev (2000) have reported five-year rolling regressions of Eq. (2) using the monthly data of DM-\$ spot and forward exchange rates and found that there are extensive periods when the rolling estimators of  $\beta$  are mostly significantly negative and the largest negative coefficient in this period is about -13.

〈Table 2〉 Test of the forward premium anomaly  
without the realized volatility and the jumps

	Euro-Dollar	Yen-Dollar
$\mu$	0.0223*** (0.0085)	-0.0023 (0.0187)
$\beta_0$	-3.2689** (1.3090)	-0.6065* (0.3685)
$\sigma^2$	0.3390*** (0.0105)	0.3351*** (0.0104)
$\ln(L)$	-1820.078	-1808.257
$m_3$	0.034	-0.348
$m_4$	3.892	4.622
$Q(20)$	11.963	7.546

Note: 1) The model is estimated by maximizing the Gaussian likelihood and the robust standard errors from QMLE are reported in parentheses below corresponding parameter estimates.

2) The quantity  $\ln(L)$  is the value of the maximized log likelihood,  $m_3$  and  $m_4$  are the sample skewness and kurtosis of the standardized residuals, and  $Q(20)$  is the Ljung-Box statistics with 20 degrees of freedom based on the standardized residuals.

3) And, the asterisks (\*, \*\*, \*\*\*) denote significance at 10%, 5% and 1% level respectively.

As pointed in Section 2, the anomaly phenomenon found in the two foreign exchange markets may be due to the problems in the regression model of Eq. (2) which did not account for both the conditional variance term of spot returns and the significant long memory property in the forward premium. So, the regression model of Eq. (2) without accounting for the problems may not be appropriate to test the forward premium anomaly and provide biased results. In order to reduce the problems in the regression model of Eq. (2), the more appropriate model based on the Eq. (5) for the tests of the forward premium anomaly could be adjusted by allowing for the realized volatility ( $RV_{t+1}$ ) and the jump process ( $\lambda$ ). The adjusted model can be specified as,

$$\Delta s_{t+1} = \alpha + \beta_0(f_t - s_t) + \beta_1 RV_{t+1} + \lambda + u_{t+1}. \quad (6)$$

This paper first uses the realized volatility ( $RV_{t+1}$ ) of the spot returns introduced by Andersen et al. (2001a) to account for the volatility term of spot exchange returns [e.g.  $\text{Var}_t(s_{t+1})$ ], in the regression model presented in Eq. (5). Thus, following Andersen et al. (2001a), the realized volatility is calculated by the summation of squared returns over the relevant horizon by using 5-minute high frequency spot exchange rates data of the Euro-Dollar and the Yen-Dollar. The high frequency 5-minute spot exchange rates are determined as the average between the bid and the ask quotes so that the 5-minute spot returns are the first differenced logarithm of the spot exchange rates and is the continuously compounded rate of return.

$$RV_{t+1} = \sum_{i=1,288} (R_{t+1,i}^2), t = 1, \dots, 2074, \quad (7)$$

where  $R_{t+1,i}$  is the 5-minute spot exchange rate return at the  $i^{\text{th}}$  interval on date  $t+1$ .

Then, in order to consider the problem of the long memory property in the forward premium, this paper employs the Bernoulli jump process which is initially proposed by Press (1967). While stochastic jumps are generally modelled by the Poisson process, the Bernoulli jump process appears to be practically more convenient in accounting for the jumps than the Poisson process since the Bernoulli process is simpler in calculation without requiring the infinite sum and the truncation process required by the Poisson process.

In the Bernoulli distribution, it is assumed that upon the arrival of economic /financial events in foreign exchange markets there is an instantaneous jump in the forward premium with a jump intensity parameter of  $\lambda$  which is defined as  $\lambda = (1 + \exp(k))^{-1}$  and is constant over time with  $0 < \lambda < 1$ . And the jump is specified to be generated by the Bernoulli distribution, and the jump size is given by the random variable  $v$  which is assumed to be  $NID(\nu, \delta^2)$  where the mean and the variance of the jumps are equal to  $\nu$  and  $\delta^2$  during a time interval. The log likelihood function for the model can be written as,

$$\ln(\hat{\xi}) = -(T/2)\ln(2\pi) + \sum_{t=1, \dots, T} \ln\{[(1-\lambda)/\sigma] \cdot \exp[-(\epsilon_t + \lambda\nu)^2/2\sigma^2] + [\lambda/(\sigma^2 + \delta^2)^{1/2}] \cdot \exp[-(\epsilon_t + (1-\lambda)\nu)^2/2(\sigma^2 + \delta^2)]\} \quad (8)$$

Table 3 summarizes the estimation results of the adjusted model with allowing for the realized volatility and the jump process in Eq. (6). For the Yen-Dollar currency markets, the estimated value of  $\beta_0$  is found to be 0.30, which is positive and statistically significant. This result suggests the possibility that the forward premium anomaly may be disappeared in the Yen-Dollar market when the regression model is appropriately specified by allowing for the jump process and the realized volatility. And, the value of

the  $\beta_0$  in the Euro-Dollar currency markets is found to be -0.47 and is statistically significant. Even though the estimated value of the  $\beta_0$  in the Euro-Dollar market is still negative but it is much lower than the value estimated from the usual model of Eq. (2) without considering the terms of the jump process and the realized volatility. Thus, even though there still exists the forward premium anomaly in the Euro-Dollar markets even when the regression model is specified more appropriately, the degree of the anomaly seems to be not as severe as before. Thus, these results of this paper lend additional support to the findings of Baillie and Bollerslev (2000) showing that the forward premium is not as bad as the previous studies have presented by using monthly data over very long spans of time.

The estimated value of  $\beta_1$  in the Yen-Dollar markets for the realized volatility in the regression model are found to be -0.39 close to the theoretical value -0.5 and is statistically significant, but the value in the case of the Euro-Dollar exchange rates is found to be 0.17 and is statistically significant. Thus, the volatility of the spot exchange returns appears to affect more significantly in the Yen-Dollar currency markets compared to the Euro-Dollar market.

Also, the estimated parameters ( $\lambda$ ) for the jump probabilities are found to be 0.54 and 0.32 for the Euro-Dollar and the Yen-Dollar currency markets respectively and they are all statistically significant at the conventional level of significance. Thus, the Bernoulli distribution appears to be quite appropriate to account for the jumps in the foreign exchange markets. And the estimated parameters ( $\nu$ ) which represents the impact of the jumps on the mean process are found to be -0.08 and 0.06 for the Euro-Dollar and the Yen-Dollar exchange rates and statistically significant at the conventional level of significance. Thus, the jumps in the Euro-Dollar markets seem to make the future spot returns decrease with appreciating

the Euro against the Dollar while the jumps in the Yen-Dollar markets may tend to increase the future spot returns with depreciating the Yen against the Dollar.

Similarly, the effects of the jumps ( $\delta^2$ ) on the volatility process are estimated to be 0.36 and 0.42 and also they are all statistically significant. It suggests that the effects of the jumps on the volatility process are also important and the effects are greater. These results present the possibility that the jumps may closely related to the unexpected movements of the exchange rates implying that the estimated Bernoulli distribution model captures the jumps appropriately in the Euro-Dollar and the Yen-Dollar currency markets.

〈Table 3〉 Test of the forward premium anomaly using the regression model with the realized volatility and the jumps

	Euro-Dollar	Yen-Dollar
$\mu$	0.0008 (0.0276)	0.1458*** (0.0383)
$\beta_0$	-0.4633* (0.2817)	0.2960* (0.1799)
$\beta_1$	0.16689*** (0.0630)	-0.3913** (0.1944)
k	- 0.1480* (0.0898)	0.7787* (0.4734)
$[\lambda]$	[0.537]	[0.315]
$\nu$	-0.0803* (0.0488)	0.0581* (0.0353)
$\delta^2$	0.3553*** (0.0380)	0.4147*** (0.1468)
$\sigma^2$	0.1450*** (0.0476)	0.1926*** (0.0525)
ln(L)	-1789.003	-1742.969
$m_3$	-0.109	0.085
$m_4$	1.379	2.219
Q(20)	13.414	11.975

Note: 1) the same as Table 2 except that a jump intensity of  $\lambda$ , where  $0 < \lambda < 1$ , and is specified to be generated by the Bernoulli process.

2) The jump size is given by the random variable  $v_t$  which is assumed to be NID ( $\nu, \delta^2$ ).

3) And, the asterisks (\*, \*\*, \*\*\*) denote significance at 10%, 5% and 1% level respectively.

## IV. Conclusion

This paper re-examines the issue of the forward premium anomaly in foreign exchange markets with and without allowing for the realized volatility and the jump process in the regression model by using the daily spot exchange rates and the overnight forward exchange rates of the Dollar-Euro and the Yen-Dollar. In particular, this paper adjusts the usual regression model allowing for the realized volatility obtained the high frequency spot exchange rates and the jumps process with the Bernoulli distribution in order to test the forward premium anomaly and then re-estimates it.

The results from the adjusted model show that the misspecification of the regression model for the tests of the forward premium anomaly without considering the volatility term of the future spot returns and the long memory property in the forward premium seems to be partly responsible for the forward premium anomaly in the foreign exchange markets. In particular, when the adjusted model allowing for the realized volatility and the jump process to reduce the problems is used for the tests of the forward premium anomaly, the estimated value of  $\beta_0$  in the Yen-Dollar exchange market is found to be positive while the value of the  $\beta_0$  in the Euro-Dollar market is found to be still negative but is much lower than the value estimated from the usual model without the jump process and the realized volatility.

Thus, this paper finds statistical evidence of the possibility that the anomalous findings may be disappeared or at least the degree of the anomaly may be decreased when the regression model for the tests of the anomaly is adjusted with allowing for the realized volatility and the jumps in order to consider the omitted spot return volatility and the long memory

property in the forward premium in the foreign exchange markets. The results of this paper have important implications for understanding the roles of the jump process and the realized volatility in foreign exchange markets because this paper can confirm the possibility that the forward premium anomaly may not be as severe as before when the test regression model is specified appropriately. Thus, the well-known forward premium anomaly may be partly a statistical artifact arising from the model misspecification of omitting the spot returns volatility and ignoring the long memory property of the forward premium in the regression model.



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## 요 약

일별 유로-달러와 엔-달러 현물환율과 확정일방 식 선물환율을 이용하여, 본 논문에서는 국제금융분야에서 많은 관심을 가지고 있는 주제인 forward premium anomaly 를 재고찰하고자 한다. 특히 본 논문에서는 기존의 논문들에서 forward premium anomaly 현상을 연구할 때 고려하지 못하였던 실현적 변동성과 점프과정의 역할에 대한 통계적 증거를 밝히고자 한다. 이를 위해 본 논문에서는 forward premium anomaly를 테스트하기 위해 사용되었던 기존의 회귀모형을 실현적 변동성과 점프과정을 도입하여 모형을 변형시킨 후 분석한 결과, 엔-달러 외환시장에서 선물프리미엄의 추정된 회귀계수 값이 양의 값으로 나타나 반면에 유로-달러 외환시장에서는 추정된 회귀계수 값이 여전히 음의 값이나 이전에 비해 매우 작은 값으로 나타남을 보였다. 따라서 본 논문에서는 회귀모형에 실현적 변동성과 점프과정을 도입하여 모형을 보다 적합하게 설정한다면, 외환시장에서 나타나는 forward premium anomaly의 경험적 현상이 기존의 논문들에서 제시하였던 만큼 심각하지 않을 수 있다는 가능성을 제시하였다.

※ 국문 색인어: 고빈도 환율, 실현적 변동성, 일별 현물환율, 확정일 방식 선물환율점프 과정, Bernoulli 분포, Forward premium anomaly