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An Exploration into the Annuity Puzzle: The Role of Health Risk, Lack of Liquidity of Annuities, and the Value of Life*

연금 퍼즐에 관한 연구: 건강 위험, 종신연금의 유동성 부족, 생존가치의 역할을 중심으로

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This paper proposes a theory that has the potential to explain the annuity puzzle, i.e., the low demand for annuities in the U.S. and other countries. Specifically, this paper finds that the lack of liquidity of annuities, combined with adverse health shocks, can account for the low demand for annuities. Annuities are less attractive because the elderly find it hard to sell back annuities in case of large medical expenses due to serious health problems. In relation to this, the paper examines the role of the value of life, an intrinsic value attached to a life itself, in the annuity puzzle. If it means a lot to be alive, people would spend more on medical care and hence could demand annuities less. In a series of quantitative analyses, however, I find that the value of life does not reduce the annuity demand much, because more medical care motivated by the value of life can increase the life expectancy, which in turn can raise the annuity demand by increasing the expected payoffs of annuities.

Key Words: Annuity puzzle, value of life, health expenditure, retirement assets
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Ⅰ. Introduction

The “annuity puzzle” has posed a big challenge to standard life-cycle theories about asset allocation with uncertain lifetimes. According to the standard studies such as Yaari (1965) and Davidoff, Brown and Diamond (2005), people should annuitize a large fraction, if not all, of their wealth in the presence of a longevity risk. In reality, however, people hardly purchase annuities and markets for private annuities in the U.S. as well as in other developed countries remain underdeveloped.

To resolve the puzzle, various theories have been proposed in the literature. Those theories include adverse selection due to the information asymmetry in life expectancy (Mitchell et al., 1999), pre-existing annuitization through Social Security (Dushi and Webb, 2004), risk sharing within family (Kotlikoff and Spivak, 1981 and Brown and Poterba, 2000), and bequest motives (Friedman and Warshawsky, 1990 and Lockwood, 2009), to name a few. General lessons from such analyses, however, are that there still remains a gap between the empirical finding and the predictions of such theories[1].

This paper contributes to the literature by proposing an alternative theory that can help to resolve the annuity puzzle. In this paper, it is the lack of liquidity of annuities and the possibility of medical expenses due to serious health problems that are jointly responsible for the low demand for annuities. Due to the possibility of serious health problems and expensive medical care, the elderly accumulate precautionary savings. In this case, they might be reluctant to annuitize retirement wealth because they find it hard to resell or liquidate annuities. For this reason, the demand for annuities can be low even though annuities provide higher returns than more liquid assets owing to the so-called mortality premium. To formulate this idea, I build a simple model of asset allocation by the elderly under adverse health shocks and the lack of liquidity of

1) For a detailed and extensive review of the literature, see Gentry and Rothschild (2006) and Brown (2007).

annuities. Then, the model is calibrated to match the data on a variety of health-related variables in the U.S. and other countries to illustrate that it can match the low annuity demand observed in the U.S..

In this paper, I find that this model indeed has the potential to account for the annuity puzzle. Specifically, in the rigorously calibrated model, retirees in the bottom three quintiles of the wealth distribution do not hold any annuities at the point of retirement. The top two quintiles do hold annuities when they retire, but the shares of annuitized wealth are only 12% and 27%, respectively, for the fourth and the fifth quintiles. These results are consistent with a couple of important empirical facts regarding the annuity puzzle. First, the demand for annuities is quite low in general. Second, wealthier retirees tend to annuitize a greater fraction of their wealth, excluding mandatory annuitization through Social Security. Such findings indicate that the lack of liquidity of annuities, together with uncertain health problems, be a potential explanation for the annuity puzzle.

This paper is particularly related to the previous studies that attempt to explain the annuity puzzle by uninsured medical expenditure. Turra and Mitchell (2004) and Sinclair and Smetters (2004), for example, show that concerns about medical expenses may lower the demand for annuities. Pang and Warshawsky (2010) and Peijnenburg, Nijman, and Werker (2010) disagree, however, arguing that if the elderly have to incur substantial medical expenses mainly at the very old ages, they may annuitize more, not less, for they can pay the expenses using benefits from annuities that offer higher returns than other assets. In this way, the impact of medical expenditure by the elderly on the annuity demand crucially hinges on the timing of the need for medical expenditure.

This paper, however, differs from such papers in a couple of ways. First, it endogenizes the decision about when and how much to spend on medical care. By contrast, most papers in the literature regard medical spending as exogenous expenditure shocks, uncorrelated with mortality. Second, this paper emphasizes the role of serious or life-threatening health problems in the demand for annuities, which has been highlighted little in the literature. It is these differences, along with the lack of liquidity of annuities, that give rise to the low demand for annuities in this paper. In other words, retirees tend to hold a significant amount of liquid assets to prepare themselves for expensive medical care required by serious health problems that may occur early in retirement. This consideration, therefore, can lead to a decline in the demand for annuities. This argument would be invalid if people can easily exchange existing annuity contracts for cash to pay the large amount of medical expenses. Therefore, the illiquidity of annuities is another important factor that lowers the demand for annuities.

Finally, this paper also contributes to the literature by examining how the introduction of the value of life, i.e., an intrinsic value placed on being alive, influences the demand for annuities. If people value their lives highly, they tend to receive more health care. Indeed, Hall and Jones (2007) show that the rising share of health-care expenditures in the U.S. GDP can be explained if people inherently place a value on being alive. This finding suggests that the value of life could have a negative effect on the annuity demand through the health expenditure channel. If this is the case, the value of life can be another explanation for the annuity puzzle.

I find, however, that the value of life does not affect the demand for annuities much because it influences the demand for annuities through two contradictory channels. On one hand, assigning some intrinsic value to a life encourages the utilization of potentially expensive medical care to raise life expectancy. The possibility of large medical expenses leads to a fall in the demand for annuities because of the illiquidity of annuities. On the other hand, such an expanded use of medical care increases expected payoffs from annuities by raising the life expectancy, which has a positive effect on the demand for annuities. It turns out that these effects almost offset each other in the quantitative analysis. I conclude, therefore, that the value of life is not a decisive factor in lowering the demand for annuities.

This paper is organized as follows. Section II sets up the model to study health expenditure and the demand for annuities in a unified framework. Section III presents the optimality conditions of the individual problem and discusses how the demand for annuities are determined. Section IV analyzes the model quantitatively by calibrating it to various health-related measures. Finally, section V concludes.

II. Model

In this section, I introduce the model to analyze the demand for annuities and medical expenditures jointly. As this paper is mainly concerned with the annuity demand by the elderly or retirees, I study the asset allocation problem of the elderly in the model. In what follows, I present the details of the model.

1. Setup

In this model, we study the asset allocation problem of old-age retirees who can live for T+1 periods, from period 0 to T. In period 0, they retire and allocate their initial wealth to an immediate life annuity and a riskless bond, which represents liquid assets. From period 1 to T, they consume and receive medical care in addition to purchasing the two types of assets. Now consider the problems of agents in detail.

1) Period 0

In period 0, agents retire and enter the model. In particular, agents with type i retire with initial wealth W^i and the entitlement to a constant stream of income y^i from period 1. Let α_i be a fraction of type i agents. They allocate the initial wealth to the annuity and bond as

$$A_1 + B_1 = W, \tag{1}$$

where A_{t+1} and B_{t+1} denote, respectively, annuity and bond holdings at the end of period t.

As for health risks, all agents are endowed with the same ex ante survival probabilities. More specifically, all agents would survive from period t-1 to t with the same probability p_t, if perfectly healthy[2]. But actual survival probabilities differ across agents since they have different histories of health problems and medical care. To distinguish actual survival probabilities from ex ante counterparts, I define π_t as the ex post or actual survival probability from period t-1 to t and discuss it in detail later. In addition to p_t and π_t, define $P_{s,t}$ and $\Pi_{s,t}$ as the ex ante and ex post survival probabilities from period s to t, for future use. Finally, assume for simplicity that no agents die before period 1, that is, $p_1 = \pi_1 = 1$.

2) Period 1 to T

Provided that agents survive in period t, each of them draws an adverse health shock $\epsilon_t \in (0,1]$ from a probability density $f_t(\epsilon_t)$. The shock represents health problems that reduce the probability of survival in period t+1. Agents with $\epsilon_t = 1$ have no such health problems in period t.

After observing the health shocks, agents receive medical care by spending m_t to improve their health. The ex post survival probability for the next period, π_{t+1}, is then determined as

2) I will formally define what it means to be perfectly healthy later.

$$\pi_{t+1} = p_{t+1}\epsilon_t x_t(m_t, \epsilon_t),$$

where x_t is the efficacy of medical care that costs m_t, given the adverse health shock ϵ_t. The formulation of π_{t+1} suggests that while health problems lower the survival probability, medical care raises it. I define perfect health as the state with $\pi_{t+1} = p_{t+1}$ or $\epsilon_t x_t(m_t, \epsilon_t) = 1$.

A few assumptions are made on the efficacy function x_t to enhance the analysis of the model. First, it is increasing and strictly concave in m_t. Second, it satisfies $x_t(0, \epsilon_t) = 1$ and $\lim_{m_t \to \infty} x_t(m_t, \epsilon_t) = 1/\epsilon_t$. The former states that health does not improve without medical care, and the latter means that $\lim_{m_t \to \infty} \pi_{t+1} = p_{t+1}$. Agents do not only receive medical care but also consume after observing ϵ_t. Hence, the budget constraint for period 1 to T is written as

$$c_t + m_t + A_{t+1} + B_{t+1} \leq y + (1 + r_t^a)A_t + (1 + r_t^b)B_t, \tag{2}$$

where r_t^a and r_t^b are net returns to annuity and bond, respectively. But for the terminal period T, $m_T = A_{T+1} = B_{T+1} = 0$ because all agents die after the period.

In addition to the budget constraints, agents are also subject to the constraints that reflect limitations in financial markets. First, it is impossible to borrow because agents could otherwise die in debt:

$$B_{t+1} \geq 0, \ \forall t \in \{0, \ldots, T-1\}. \tag{3}$$

Second, it is also restrictive to reduce annuity holdings:

$$A_1 \geq 0, \ A_{t+1} \geq \delta A_t, \ \forall t \in \{1, \ldots, T-1\}, \tag{4}$$

where $0 \leq \delta \leq 1$. The equation reflects the fact that it is very difficult for individuals to resell annuity contracts for cash. This constraint makes annuities less useful to pay a large amount of medical expenses in case of adverse health shocks. The parameter δ measures how restrictive resales of annuities are. If $\delta = 1$, agents are now allowed to liquidate the existing annuity at all, whereas if $\delta = 0$, they can freely do so. In the quantitative analysis of the model in section IV, I will take $\delta = 1$ as the baseline case and examine how the demand for annuities varies with the parameter.

3) Preferences and the Value of Life

In this model, I adopt the following utility function.

$$U = \begin{cases} u(c) + v, & \text{if } alive \\ 0, & \text{if } dead \end{cases}$$

The first term $u(c)$ is standard with $u' > 0$ and $u'' < 0$. The second term v represents the value of life, which is the value that people place on being alive in this model. The formulation is similar to the baseline specification of Hall and Jones (2007) to account for a rising share of the health care spending in U.S. GDP. With the additional v term, people are likely to receive more medical care to extend their lives.

Why would people value the life in itself? One possible answer is that death is arguably the biggest uncertainty in life because there is absolutely no information on what follows the death or how it feels to die. In case of other types of risks such as disability or unemployment, however, people can have an idea about the impact of such risks. By contrast, it is impossible to get any idea on the effect of death, let alone the level of utility after death. For this reason, people are likely to feel great uncertainty about death. Hence, it seems reasonable to introduce a utility difference between being dead and being alive. Introducing the value of life v is one way to model this utility difference.

2. Agents' Problem

In this subsection, I formulate the problem of agents. In each period, they make decisions based on the history of health shocks $(i, \epsilon_1, \cdots, \epsilon_t)$, denoted by ϵ^t. The history occurs with the unconditional probability $\Pr(\epsilon^t)$, calculated as

$$\Pr(\epsilon^t) = \alpha_i \prod_{j=1}^{t} f_j(\epsilon_j).$$

Now let us write down the agents' problem. An agent with a type i maximizes the expected utility

$$\sum_{t=1}^{T} \beta^t \sum_{\epsilon^t} \Pr(\epsilon^t) \Pi_t(\epsilon^{t-1}) \left[u(c(\epsilon^t)) + v \right], \tag{5}$$

subject to the budget constraints (equations (1) and (2)), as well as the borrowing constraint (equation (3)) and the resale constraint for the annuity (equation (4)). Notice that $\Pi_t(\epsilon^{t-1})$ is endogenously determined in period t-1. In the remainder of the paper, I will write equation (5) succinctly as

$$E_0 \left[\sum_{t=1}^{T} \beta^t \Pi_t \{ u(c_t) + v \} \right],$$

where the expectation is taken over the history ϵ^t.

III. Optimal Allocation and the Demand for Annuities

In this section, I characterize the optimal allocation of the agents' problem and discuss why the annuity demand can be low in this model. All conditions presented in this section are obtained from the first-order conditions of the agents' problem.

1. Consumption and Medical Expenditure

The allocation between consumption and medical expenditure is governed by

$$u'(c_t) = E_t\left[\sum_{s=t+1}^{T} \beta^{s-t} \frac{\partial \Pi_{t,s}}{\partial m_t} \{u(c_s) + v\}\right], \tag{6}$$

where the expectation is again taken over the history of the health shocks ϵ_t. This condition suggests that the marginal utility of consumption should be equated to the marginal benefit from medical expenditure. If an agent receives a medical treatment by sacrificing a unit of consumption, she will improve her health, which, in turn, will raise the probability of survival $\Pi_{t,s}$ through a positive effect on π_{t+1}.

Decision-making on medical expenditure is associated with the adverse health shocks. To investigate the response of m_t to ϵ_t, it is useful to rewrite equation (6), using the chain rule, as

$$u'(c_t) = \frac{\partial x_t(m_t, \epsilon_t)}{\partial m_t} \epsilon_t p_{t+1} E_t\left[\sum_{s=t+1}^{T} \beta^{s-t} \Pi_{t+1,s} \{u(c_s) + v\}\right]. \tag{7}$$

This expression is convenient since m_t only appears in $\partial x_t(m_t, \epsilon_t)/\partial m_t$ and all other terms are independent of m_t. Given this condition, individuals raise m_t in response to an adverse health shock $\epsilon_t < 1$, if the following condition holds:

$$\frac{\partial x_t(m_t, \epsilon_t)}{\partial m_t} + \frac{\partial^2 x_t(m_t, \epsilon_t)}{\partial m_t \partial \epsilon_t} \epsilon_t < 0. \tag{8}$$

A necessary condition for equation (8) is $\frac{\partial^2 x_t(m_t, \epsilon_t)}{\partial m_t \partial \epsilon_t} < 0$. It means that medical care improves health more for agents with poorer health. If this effect is strong enough, people with more serious health problems spend more on medical care. When I calibrate the model in section IV, I will use a functional form for x_t that satisfies equation (8).

2. Annuity and Bond

The condition that governs asset allocation between annuity and bond is

$$\beta\left(r^a_{t+1} - r^b_{t+1}\right)\pi_{t+1}E_t\left[u'(c_{t+1})\right] = (\mu^B_t - \mu^A_t) + \beta\delta\pi_{t+1}E_t\left[\mu^A_{t+1}\right], \tag{9}$$

where μ^A_t is a multiplier on the condition $A_{t+1} \geq \delta A_t$ (equation (4)) and μ^B_t is a multiplier on the condition $B_{t+1} \geq 0$ (equation (3)). Equation (9) is obtained from combining the intertemporal Euler equations of the two assets. It reveals two factors that determine the allocation of assets: the return to each type of asset and the inflexibility of annuity. Without the inflexibility of annuity, asset returns would be solely responsible for the asset allocation. In fact, one can see from equation (9) that if $\delta = 0$, agents would only purchase the asset that offers a higher return. For example, $r^A_{t+1} \geq r^B_{t+1}$ implies a positive μ^B_t, or equivalently, $B_{t+1} = 0$.

In this model, however, it is restrictive to exchange annuities for cash, as indicated by $\delta > 0$. Hence, in this model, the annuity is less attractive than the bond in the sense that even if the annuity provides a higher return than the bond, agents may still hold bonds as well as annuities. Algebraically, even if $r^a_{t+1} > r^b_{t+1}$, it is still possible that $\mu^B_t = 0$ or $B_{t+1} > 0$ due to the term $\beta\delta\pi_{t+1}E_t\left[\mu^A_{t+1}\right]$ in equation (9). Intuitively, people only purchase the bond if $r^a_{t+1} < r^b_{t+1}$ because it is both more profitable and more liquid. In contrast, even if $r^a_{t+1} > r^b_{t+1}$, they may hold the bond because it is more liquid than the annuity. In this model, therefore, the annuity demand is expected to be lower than it would be under $\delta = 0$.

The lack of liquidity of the annuity would not matter much without the adverse health shocks. With the possibility of the large adverse health shocks, however, the constraint that annuity holdings cannot be lowered below a certain level is likely binding in the state with a serious health problem (a low level of ϵ_{t+1}), making μ_{t+1}^{A} positive. This leads the term $\beta\delta\pi_{t+1}E_t\left[\mu_{t+1}^{A}\right]$ in equation (9) positive as well. In this case, it is also possible that $\mu_t^A > 0$, even if $r_{t+1}^{a} > r_{t+1}^{b}$. Therefore, the possibility of the adverse health shocks in the future that require a large medical expenditure can account for the low demand for annuities in the current period.

To conclude, the annuity is less popular in this model because it cannot be traded easily for cash when adverse health shocks necessitate large medical expenses. To be a proper explanation for the annuity puzzle, however, this model should be able to generate sufficiently low demand for annuities. As this is a quantitative question, I test the quantitative relevance of this model for the annuity puzzle by calibrating it in the next section. I show that the annuity demand is indeed very low in the model calibrated to match the data on a variety of health-related measures in the U.S. and other countries.

Ⅳ. Quantitative Analysis of the Demand for Annuities

In this section, I calibrate the model and describe the results. I begin with choosing parameter values for the calibration.

1. Parameterization of the Model

1) Basic Parameters

I adopt the log utility function augmented by the value of life term:[3)]

$$U(c;v) = \ln c + v.$$

The interest rate on the risk-free bond r^b is set at 3% per annum. Discount factor β is assumed $1/(1+r^b)$. As this model is concerned with the asset allocation of the elderly, I consider a 40-year period from age 65 to 104. The unit model period is chosen to be 10 years for computational simplicity, which implies that the terminal period T=4. Period 0 corresponds to the beginning of age 65, when agents in the model make a choice on asset allocation.

In this model, I consider an immediate life annuity that offers a stream of real payments until death. The annuity return r_t^a is set to make the annuity actuarially fair for the perfectly healthy. Though it is then actuarially unfair for the average people, the money's worth of the annuity calculated with results of the simulation still exceeds 90%. Hence, this can be regarded as a convenient way to introduce the actuarial unfairness of annuities to this quantitative analysis. Finally, I set $\delta = 1$, which means people cannot decrease annuity holdings at all. This assumption will be relaxed later to highlight the role of δ.

As for the value of life, I set the value of additional year of life to be 100,000 dollars. This has been widely used in health economics literature (Culter, 2005, for example), though the measured value of life varies a lot with the context in which it is estimated[4]. With the unit period equivalent to ten years, I will set v as the consumption utility of million dollars in the model.

3) In the earlier version of the paper, I used a general CRRA (constant relative risk aversion) utility function and showed that the annuity puzzle could be even better accounted for if CRRA〉1. Thus, I only consider the case with CRRA=1.

4) Refer to Ashenfelter (2006) and Viscusi (2008), for details.

2) Parameters Related to Health and Mortality

To begin with, I assume the adverse health shock ϵ_t is binary as

$$\epsilon_t = \begin{cases} 1, & with\,prob\ 1-\psi_t \\ d_t, & with\,prob\ \psi_t \end{cases}$$

where $d_t \in (0,1)$ is interpreted as a relative decline in the survival probability due to diseases. A state with $\epsilon_t = 1$ represents the absence of such diseases in period t. To calibrate ψ_t, I need to determine the types of health problems that ϵ_t represents. I regard heart diseases, cancer, and strokes collectively as the health shock in the model as they are the most important causes of death in the U.S. Then, ψ_t is set to match the proportion of people who are newly diagnosed with those diseases for age group t in the Health and Retirement Study (HRS)[5]. Though this model considers a limited set of health problems, it can still manage to match the observed low demand for annuities quantitatively, as we will see.

To calibrate d_t, I use the relative survival rates for the diseases represented ϵ_t. It is defined as the ratio of a survival rate of people with a certain disease to that without the disease. Let Q_{t+1} denote the relative survival rate from period t to $t+1$ for the diseases in this model. It is computed in the model as

$$Q_{t+1}(\epsilon^t) = \frac{\pi_{t+1}(\epsilon^{t-1}, d_t)}{\pi_{t+1}(\epsilon^{t-1}, 1)} = \frac{d_t x_t(m_t(\epsilon^{t-1}, d_t), d_t)}{x_t(m_t(\epsilon^{t-1}, 1), 1)},$$

where the history (ϵ^{t-1}, d_t) indicates the history ϵ^t with $\epsilon_t = d_t$ and $(\epsilon^{t-1}, 1)$ with $\epsilon_t = 1$.

Now consider a person who is too poor to afford any medical care. For the person,

5) Recall that $\psi_4 = 0$, as $T = 4$.

$m_t(\epsilon^{t-1}, d_t) = m_t(\epsilon^{t-1}, 1) = 0$, which implies $Q_{t+1}(\epsilon^t) = d_t$ regardless of history ϵ^{t-1}, as $x_t(0, \epsilon_t) = 1$ for any ϵ_t. While it is extremely difficult to find income- or wealth-specific relative survival rates in the U.S., the relative survival rates for underdeveloped countries may be a reasonable proxy for them. Coleman et al. (2008) compile the five-year relative survival rates for major types of cancers in various countries. I take values for Algeria, one of the most underdeveloped countries in their sample. Given that the unit period in the model is ten years, it would be desirable to have a ten-year relative survival probability of cancer patients in Algeria. However, relative survival rates of cancer patients do not decline much after a couple of years since diagnosis (Brenner, 2002). Consequently, five-year relative survival rates can reasonably approximate the ten-year counterparts needed for our simulation. Finally, owing to the lack of data availability, I make a simplifying assumption that d_t is constant over time. Hence, I set $d_t = 0.2392$, which is a simple average of the five-year relative survival rates for different types of cancers in Algeria.

To pin down the $\{p_t\}_{t=1}^{T}$ sequence, I use the information about survival probabilities of the population and relative survival rates for the diseases of interest in the U.S. Let $\overline{\pi}_{t+1}$ denote the average ex post survival probability from t to t+1. I use the U.S. life table (Arias et al., 2008) to find the $\{\overline{\pi}_t\}_{t=1}^{T}$ sequence. From the viewpoint of this model, $\overline{\pi}_{t+1}$ should satisfy

$$\overline{\pi}_{t+1} = \frac{\sum_{\epsilon^t} \Pi_t(\epsilon^{t-1}) \Pr(\epsilon^t) \pi_{t+1}(\epsilon^t)}{\sum_{\epsilon^t} \Pi_t(\epsilon^{t-1}) \Pr(\epsilon^t)}, \tag{10}$$

where $\Pi_t(\epsilon^{t-1}) \Pr(\epsilon^t)$ is a fraction of people with the ex post survival probability $\pi_{t+1}(\epsilon^t)$. Since ψ_t is independent of the history up to period t-1 and $\pi_{t+1} = p_{t+1}$ if $\epsilon_t = 1$, equation (10) can be simplified to

$$\overline{\pi}_{t+1} = p_{t+1}\left[(1-\psi_t)+\psi_t\overline{Q}_{t+1}\right], \tag{11}$$

where $\overline{Q}_{t+1}$ is the average relative survival rate, defined as

$$\overline{Q}_{t+1} = \frac{\sum_{\epsilon^{t-1}} \Pi_t(\epsilon^{t-1})\Pr(\epsilon^{t-1})Q_{t+1}(\epsilon^t)}{\sum_{\epsilon^{t-1}} \Pi_t(\epsilon^{t-1})\Pr(\epsilon^{t-1})}.$$

I calibrate p_{t+1} using equation (11). Suppose I select a value for p_{t+1}, which effectively determines $\overline{Q}_{t+1}$ because both ψ_t and $\overline{\pi}_{t+1}$ are already pinned down. Given p_{t+1} and other parameters, the model can be solved to give rise to another $\overline{Q}_{t+1}$ as an outcome. Let $\overline{Q}_{t+1}^{MODEL}$ denote the average relative survival probability that the model generates. $\{p_t\}_{t=2}^{T}$ is calibrated to make $\overline{Q}_{t+1}^{MODEL}$ coincide with $\overline{Q}_{t+1}$ according to equation (11), which is required for internal consistency of the model. Once determined, the values for $\{p_t\}_{t=2}^{T}$ are maintained throughout the quantitative analysis, for clear comparisons.

Finally, the efficacy of medical care should be calibrated. Since a general functional relationship between medical expenses and health improvements is not available in the literature, I use a functional form of x_t that has useful properties discussed in section II. Specifically, I assume that

$$x_t = \frac{1}{\epsilon}\left[1-\frac{\gamma}{m+\gamma}(1-\epsilon)\right], \quad \gamma > 0.$$

One can easily verify that this function has the properties proposed in section II, including equation (8) needed for expenditure m to rise as ϵ falls. γ is the parameter which governs the efficacy of medical care: the higher γ is, the less effective is the medical care. I choose γ so that the simulated model can match $\overline{Q}=0.55$, the average 10-year relative survival rate of cancer patients in the U.S. (Brenner, 2002, for example).

3) Parameterization of Wealth and Income

I assume that the wealth-income pair (W^i, y^i) is uniformly distributed over five wealth-income groups. In order to obtain (W^i, y^i) for $i = 1, \cdots, 5$, I use the variable "total wealth including second residence," at the age of 64 for single households, in the RAND HRS data. To obtain a real wealth measure, I deflate the wealth and income by the Consumer Price Index (100 in 2000). Then, I take the median wealth for each quintile as a measure of initial wealth in the model. Then, to obtain a non-asset income for each wealth quintile group, I compute the median of the average real income, excluding one from annuities, since age 65, for each wealth quintile. I use the resulting value as y^i for wealth quintile i in the model. This concludes the parameterization of the model for calibration and the parameter values explained so far are summarized in Tables A1 and A2 in the Appendix.

2. Calibration Results

1) The Demand for Annuities

I first examine the annuity demand in the calibrated version of this model. In particular, I investigate whether this model can quantitatively match two important stylized facts in the literature: the annuity demand is in general very low and wealthier retirees tend to annuitize more of their wealth than those less wealthy. I will show that this model is indeed capable of accounting for these facts quantitatively.

To investigate the demand for annuities, consider Table 1, which reports the amount of annuity and total wealth for the first two model periods or at the ages of 65 and 75.

Focusing on the asset allocation upon retirement (at age 65), we can make two observations. First, the annuity demand is indeed very low. It is either zero or very low at 12.2% and 26.5%, respectively for the fourth and fifth quintiles of the wealth distribution. Moreover, the share of annuity in wealth rises with wealth: The wealthier agents hold a greater fraction of annuitized wealth. These findings are clearly consistent with the aforementioned stylized facts.

Though agents do not hold annuities much upon retirement, they may buy significantly more annuities during the retirement. If this is the case, then part of the annuity puzzle would remain unresolved in this model. To investigate this possibility, let's look at the asset allocation at the age of 75 in Table 1. Notice that the asset allocation depends on the adverse health shock that hits agents between ages 65 and 74, or in t=1. For each wealth quintile, agents with health problems or $\epsilon_1 = d$ cannot afford to buy additional annuities at all, due to medical expenditure. Rather, they allocate all of their disposable wealth to liquid bond in preparation for future health shocks. In contrast, agents without health problem or $\epsilon_1 = 1$ can raise the annuity holdings moderately as long as they have wealth. Medical care is unnecessary for them and they have enough wealth to set aside for annuity. As at age 65, the new annuity purchases for agents at age 75 without the health shock rise with wealth. Regardless of the health shocks, however, additional purchases of the annuity are either zero or moderate. While the annuity-wealth ratio rises significantly at age 75, it is not so much due to the rising annuity holdings as due to the depleting wealth. Taken together, these findings are in line with the stylized empirical facts related to the annuity demand.

Table 1 only presents asset allocation for ages 65 and 75 in the calibrated model. Asset allocation after age 75 is not reported because annuity holdings rarely change after age 75. The intuition behind this result is that, given adverse health shocks, agents' priority is to accumulate liquid assets for possible medical expenses. Then gradually depleting wealth over the retirement would make annuity even less affordable. Consequently, most agents maintain the annuity holdings determined at age 75. To illustrate the annuity-wealth dynamics, Figure 1 displays the dynamics of annuity and wealth for the perfectly healthy. As they do not incur any medical expenditure during retirement, they are expected to hold the largest amount of annuity. According to Figure 1, though, even this type of agents does not buy additional annuities after age 75, mostly because of depleting wealth. The result suggests that agents tend to maintain the annuity holdings at age 75 for the rest of their lives, since annuity is hardly affordable to any agent.

To conclude, the quantitative results in Table 1 and Figure 1 suggest that the calibrated model can match the key empirical facts on the annuity puzzle. Specifically, both empirically and in this model, the annuity demand is quite low in general and the annuity-wealth ratio tends to rise with agents' wealth. As discussed in section III, the lack of liquidity of annuity, uncertain health shocks, and the value of life could be responsible for the quantitative findings in this model. Thus, I discuss their quantitative effects on the annuity demand in the remainder of this paper. I begin with medical expenditure through which adverse health shocks affect the annuity demand.

2) Medical Expenditure

Medical expenditure and health shocks are crucial to the key quantitative results of this model described so far. Hence, I discuss important findings on medical expenditure in Table 2 that reports average medical expenditures over all agents and over each quintile of wealth distribution. First of all, it is notable that the average medical expenses decline with age for all wealth quintiles. From the model's perspective this is perfectly natural, as younger agents enjoy greater benefits from medical care. Recall the discussion associated with equation (6). By receiving medical care in period t, agents can raise the probability of reaching future periods through a rise in π_{t+1}. Thus, this benefit decreases with t and agents tend to spend more on medical care when they are younger, as in Table 2.

The declining pattern of medical expenses plays a key role in accounting for the annuity puzzle. As the elderly should be prepared for a large medical expenditure early in retirement, they prefer liquid assets to the annuity. We can appreciate this result better if we recall that the adverse health shocks represent serious health problems such as cancer, heart diseases, and strokes that can cause death. Thus, this model suggests that the elderly do not want to annuitize wealth because they are worried about serious health problems that may occur to them early in retirement. This mechanism helps resolve a challenge in the literature on the relationship between annuities and medical expenditure. In the literature, it is shown that the medical expenditure can explain the annuity puzzle only if it is needed early in retirement. In this model, retirees tend to spend more on medical care early in retirement because the serious health problems can occur to them quite early. It is one of the key reasons that this model can generate the very low demand for annuities.

The medical expenditure declining over time makes sense in this model. But it seems at odds with the well-documented fact that medical expenditure tends to increase with age (Di Nardi, French, and Jones, 2010, e.g.). Two explanations might be helpful in reconciling the empirical fact and the model prediction on medical expenditure. First, the set of health problems in this model is restrictive than what the elderly actually face. In particular, this model abstracts from the long-term care which is mainly responsible for the rising pattern of medical expenses. In this sense, this paper can be viewed as complementing the previous studies on medical expenditure rather than as contradicting them. Second, the ex ante survival probability, $\{p_t\}_{t=1}^{T}$, may have taken away health problems that may require additional medical care and related expenditure. As people age, their bodies are likely to become weaker and that is why the elderly need more health care and their survival probabilities tend to be low. By setting a fraction of survival probability exogenous, this model does not account for the medical expenditure due to health problems related to general body weakness of the elderly. Hence, this model's prediction on medical expenditure could be justified, if we take these explanations into consideration.

Fortunately, however, this model can match another well-established finding in the literature that medical expenses increase with wealth (Di Nardi, French, and Jones, 2010, e.g.). We can see this pattern in Table 2. In this model, the wealthy can enjoy a high level of consumption and utility as long as they are alive. Hence, they would try to extend their lives longer than the less wealthy by spending more on medical care. Thus, the positive correlation between wealth and medical expenditure naturally arises.

3) Effects of the Value of Life

The value of life in the agents' preferences translates into a strong incentive for increasing life expectancy. Hence, agents with the value of life are expected to spend more on medical care, which could contribute to lowering the demand for annuities. To assess the quantitative relevance of this effect, I calibrate the model without the value of life or $v = 0$ and compare the calibration results with those with the value of life $v > 0$.

Table 3 summarizes the comparison between the models with and without the value of life. Without the value of life, agents do not attach any intrinsic value to being alive and they only care about consumption. While this change causes large falls in medical expenses, it does not influence the annuity demand much. The effect of v on average medical expenditure is intuitive because people without the value of life would not spend excessively to increase their life expectancy. Observing this effect, one might expect that the annuity demand should be higher without the value of life, for the inflexibility of annuity would not matter much for agents who intend to spend little on medical care. According to calibration results, the annuity demand indeed rises as v falls but the impact is quantitatively negligible. How does this happen?

To understand that, notice the big drops in the relative survival rates ($\overline{Q}$ or $\overline{Q}_t$ in the model) in Table 3, due to the low utilization of medical care. These changes lead to falls in survival probabilities $\{\pi_{t+1}\}$ as well. This, in turn, makes the annuity less attractive by reducing the expected sum of payments from the annuity. In sum, the elimination of v, the term representing the value of life, has two contradicting effects on the demand for annuities. On one hand, it may result in a rise in the annuity demand because the lack of liquidity matters less. On the other hand, it may lead to a fall in the annuity demand because the annuity provides less incomes on average. In this model, two contradictory effects almost offset each other, generating the insignificant rise in the annuity demand when the parameter reflecting the intrinsic value of life is eliminated.

3. Discussion

1) The Lack of Liquidity of Annuities

One of the key features that contribute to quantitative findings of this model is the lack of liquidity of annuities, captured by the parameter δ. To highlight the role of δ quantitatively, I display the effect of δ on the annuity-wealth ratio upon retirement or A_1/W in Figure 2. As δ declines, the annuity-wealth ratio rises monotonically for all quintiles with positive wealth, confirming the prediction of the model. Moreover, the threshold value of δ that enables full annuitization rises with wealth. For example, the most wealthy agents would annuitize fully even with $\delta = 0.4$, but agents in the second quintile would do so only with $\delta = 0$. Less affluent retirees are less tolerant for the inflexibility of the annuity because they do not have sufficient wealth to finance medical expenditure in case of adverse health shocks. Thus, even a small degree of illiquidity prevents them from buying the annuity. This result is comparable to the finding that the annuity wealth ratio tends to increase with wealth.

Figure 2 also shows that all agents would fully annuitize all wealth if annuity were fully liquid or $\delta = 0$. In this case, agents would not have any problem reducing annuity holdings and therefore they would only purchase annuity since $r_t^a > r_t^b$ in this model. This result obviously confirms the standard result in the literature that the full annuitization is optimal.

2) Implications of the Model

So far, we have seen that the calibrated model can quantitatively match several empirical facts in the literature on the annuity puzzle and, to some extent, on medical expenditure. In particular, the annuity demand is low in general but rising in wealth. These findings are mainly due to the uncertain health shocks and annuities' lack of liquidity. More specifically, with uncertain health shocks that may require a large amount of medical expenditure, agents are reluctant to purchase annuities that cannot easily be traded for cash.

An implication of this model is that the annuity demand should be closely related to the demand for health insurance. If retirees have health insurance that can significantly reduce out-of-pocket medical expenses, they do not worry much about future medical expenses. Hence, they purchase annuities more than they otherwise would. According to this model, therefore, we expect a positive correlation between the annuity holdings and measures of health insurance coverage at individual level, such as the number of health insurance contracts, the coverage of medical conditions, and the amount of insurance premium. One could test whether the positive correlation is actually observed in the data to judge the validity of the theory proposed in this paper.

Korea could provide an environment in which this theory is adequately tested. In the last decades, Korea's government has expanded the set of medical conditions covered by the Nation Health Insurance. Moreover, a number of private health insurance products have been newly introduced and widely utilized across people. These changes have made most people better insured against medical expenditure shocks. In This case the theory in this paper predicts that people should hold more annuities than before. If the panel data that contains detailed information on the annuity holdings and health insurance coverage is available, the model's implication could be tested. Though I do not attempt such a test in this paper, a future study on complementarity between annuities and health insurance will be certainly worthwhile.

This paper also has a couple of policy implications. Since annuities ensure a basic level of consumption during retirement regardless of the lifespan of people, government could improve welfare by inducing people to hold annuities more. This issue will be particularly important in foreseeable future as the National Pension System will not provide a sufficient level of insurance against the longevity risk. Thus, it is now almost essential for government to have people purchase more private annuities. This paper offers a couple of suggestions in this regard. First, government should pursue a policy to enhance the liquidity of annuities. The key to the annuity puzzle is the lack of liquidity of annuities, according to this paper and the calibration of the model shows that a meaningful decline in δ could attract many people to annuities. Second, the health insurance system, both public and private, should develop together with the annuity markets. As already discussed, more people can be willing to buy more annuities if they are well insured against medical expenditure shocks.

V. Conclusion

This paper analyzes how annuities' lack of liquidity, together with adverse health shocks and the value of life, can account for the annuity puzzle. I show, by a series of quantitative analyses, that the model with aforementioned factors can generate the significantly low demand for annuities through the medical expenditure channel. Also, I find that the inflexibility of annuities is likely the main culprit for the lack of interest in annuities. Thus, people may demand annuities more, if they are allowed to liquidate a fraction of annuity contracts they hold. In contrast, introducing the value of life to the agent's preference does not matter much for the demand for annuities since it influences the demand for annuities through two offsetting channels.

This paper could be refined in a couple of ways. First, this paper can be improved by utilizing data on medical spending and its efficacy, as their functional relationship can be investigated more rigorously. Second, one can extend this model by broadening the set of diseases that matter. While I choose cancer, heart diseases, and strokes as the adverse health shocks because they are the most important causes of death in the U.S., they hardly represent all health issues that the elderly consider to determine the demand for annuities. Nonetheless, this paper can be a useful starting point to investigate the demand for annuities and the relation between health risk that requires medical care and mortality jointly. Future work that incorporates more information on health issues to this model will certainly be helpful to understand the annuity puzzle better and its relation to medical expenditure.

Appendix: Model Parameterization

〈Table A1〉 Baseline parameterization for the simulation

parameter	description	value
T	terminal period	4
σ	relative risk aversion	1
δ	index of market incompleteness	1
β	utility discount factor of agents	1/(1+0.03)
r^b	annual return to risk-free bond	0.03
r_t^a	annual return to annuity	{0.03,0.0552,0.0992,0.2189}
v	value of life	3.12
ψ_t	probability of adverse health shock	{0.0754,0.0964,0.1160,0}
$d_t = d$	decline of health due to the shock	0.2392
p_t	ex ante survival probability	{1,0.7869,0.5238,0.1836}
γ_t	efficacy of medical care	1.13

〈Table A2〉 Distribution of the initial wealth and income

(Unit: U.S. dollars in 2000)

quintile	initial wealth (W^i)	non-asset income (y^i)
1	0	7,502
2	6,607	10,080
3	55,820	10,891
4	153,101	12,162
5	441,582	13,198

〈Table 1〉 Asset allocation for ages 65 and 75

(Unit: thousand dollars in 2000, percent)

wealth quintile		1		2		3		4		5	
age 65 (t=1)	annuity	0		0		0		18.7		117.1	
	wealth	0		6.6		55.8		153.1		441.6	
	ratio (%)	N/A		0		0		12.2		26.5	
health shock for 65-74 (ϵ_1)		1	d	1	d	1	d	1	d	1	d
age 75 (t=2)	annuity	0	0	1.8	0	26.1	0	63.6	18.7	178.2	117.1
	wealth	0	0	1.8	0	26.9	0	86.6	18.7	265.8	153.2
	ratio (%)	N/A	N/A	100	0	97.0	N/A	73.5	100	67.1	76.4

Note: Health shock for ages 65-74 refers to the adverse health shock ϵ_1 in the model. $\epsilon_1 = 1$ indicates that the agent remains perfectly healthy and $\epsilon_1 = d$ means that health worsens.

〈Table 2〉 Average medical expenditure by age and quintile

(Unit: dollars in 2000)

age	65–74	75–84	85–94
population	6,541	4,283	2,182
quintile 1	1,778	1,016	0
quintile 2	2,978	2,010	0
quintile 3	5,108	3,033	1,049
quintile 4	8,323	5,187	2,825
quintile 5	14,516	10,022	6,762

Note: Averages are taken over relevant histories of health shocks.

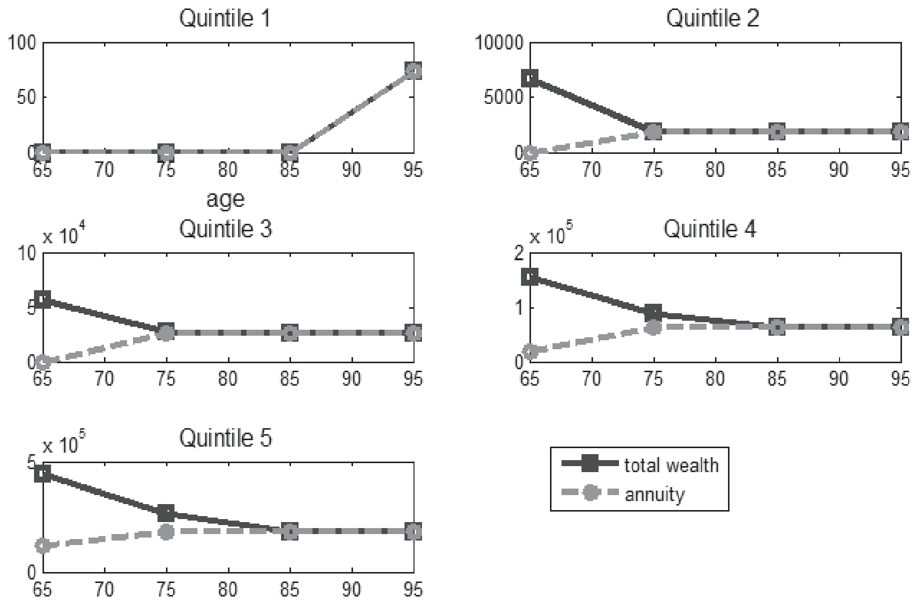
〈Table 3〉 Effects of the value of life

(Unit: percent, dollars in 2000)

model specification		with the value of life	without the value of life
annuity-wealth ratio upon retirement by wealth quintile (%)	1	N/A	N/A
	2	0	0
	3	0	0
	4	12.18	13.60
	5	26.51	28.97
average relative survival rate by age (%)	overall	55	38
	75	67	46
	85	55	36
	95	40	31
average survival probability by age (%)	75	78	77
	85	47	45
	95	14	13
average medical expenditure by age (dollars in 2000)	65-74	6,541	2,632
	75-84	4,283	1,523
	85-94	2,182	827

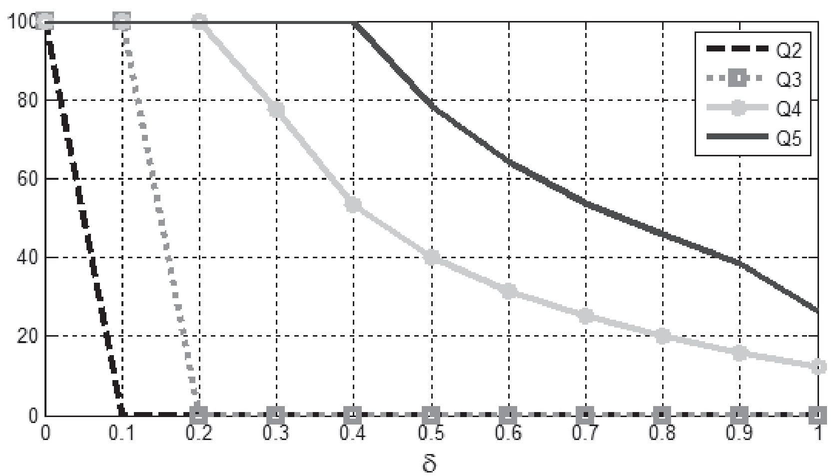
〈Figure 1〉 Annuity and wealth for the perfectly healthy

(Unit: dollars in 2000)



〈Figure 2〉 The annuity–wealth ratio upon retirement by δ

(Unit: Percent)



Note: $\delta \in [0,1]$ is an index on difficulty associated with reducing annuity holdings. If $\delta = 1$, it is impossible to lower annuity holdings over time. If $\delta = 0$, there is no restriction in doing so.

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요약

본 논문은 연금 퍼즐, 즉 미국과 다른 나라들에서 관찰된 낮은 종신연금 수요를 설명할 수 있는 이론을 제시한다. 구체적으로 본 논문에서는, 종신연금의 유동성이 부족하다는 한계점이 건강과 관련한 충격과 복합적으로 작용해 종신연금에 대한 낮은 수요를 설명할 수 있음을 보인다. 즉, 은퇴자들은 심각한 건강 문제로 인한 고액의 의료비를 지출해야 할 경우에도 종신연금을 현금화하기 어렵다는 점 때문에 종신연금을 선호하지 않는다. 또한 본 논문에서는 연금 퍼즐과 관련해, 생존가치, 즉 살아있는 것 자체에 두는 가치의 역할을 검토한다. 만약 살아 있는 것에 사람들이 큰 의미를 둔다면 더 많은 의료비를 지출할 것이고, 그 결과 종신연금을 덜 수요하게 될 수 있다. 그러나 이와 관련한 정량적 분석에서 생존가치를 도입해도 종신연금 수요가 그다지 감소하지 않는 것으로 나타났다. 그 이유는 생존가치 때문에 더 많은 의료 혜택을 받아 기대 여명을 높일 수 있고, 그 결과 종신연금의 기대수입을 높여 종신연금 수요를 증대시킬 수 있기 때문이다.

※ **국문 색인어:** 연금퍼즐, 생존 자체에 대한 가치, 의료비, 은퇴 자산