

대학 수학의 미분법을 위한 Maple 활용

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요약

대학에서의 미분법 강의는 기본 개념을 이해하고 문장이나 기호로 주어진 미분법 문제를 해석 할 수 있도록 하는 데 초점이 맞춰져있다. 본 논문의 목적은 Maple을 활용한 효과적인 미분지도 방안에 관한 연구이다. 수학용 프로그램의 하나인 Maple로 다양한 함수들을 표현하고, 미분의 개념을 설명하는 데 Maple을 사용하였다. 그리고 삼각함수의 고계도함수를 구하는 방법을 알아보았다.

이 논문은, 미분법을 학습하고 있는 학습자가 학습한 내용을 단순한 알고리즘의 습득이 아니라, 수학적 탐구의 기회로 활용하기 위한 방법을 제시하였다.

Maple application of university mathematics for differentiation

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ABSTRACT

Lectures differentiation at universities have focused on that to be able to understand the basic concepts, to interpret the problem differentiation method given in symbols or sentences. The purpose of this thesis is studying an efficient teaching method for differentiation using Maple. Maple was utilized to explain the concept of differentiation and to express various differentiation. And we treat differentiation of functions including trigonometric functions.

In this paper, differentiation in learning and learner information, learning as the acquisition of a simple algorithm, mathematical explore ways to take advantage of the opportunities presented.

Key Words : Differentiation, Computer Assisted Instruction, mathematical program, Maple

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I. Introduction

Traditionally, basic courses like Liberal Arts focus on the understanding of foundations and the solution with mathematical symbols. Differential calculus, a required course for the Department of science and engineering, is the study of advanced mathematics. When problems in the lesson were depicted through mathematical symbolization they could be solved using simple calculation. Assignments had been focused on gaining the algorism.

At Mathematics Instructors conference in America in the 1990s, they admitted in "Curriculum and evaluation standards for school mathematics" that a calculator and a computer should be accepted as a teaching tool to improve the quality of Mathematics education.

The 7th Math Curriculum in Korea, published in 2000 writes "To improve capability of problem-solving and the understanding of Mathematical concept, principle and laws, using a calculator and a computer is recommended as much as possible except the arena for developing calculation capability." It emphasized creativity development and social adaptability through learning with the use of computers in math education. As a result of distribution of the computers, there have been many changes in studying differentiation in University Mathematics courses. Learners did not need to consider the level of problems or complexity. Using the symbols and solving the problems are different from understanding the meaning. The concepts in differentiation such as limit in function, continuation,

differential possibility, and rate of change are more understandable through the Quantitative Approach. This approach improves intuitive ability and helps first-time learners understand difficult concept better.

Maple is a program for studying Mathematics. Maple can be used as a tool for practicing and studying the mathematical fact or principles depending on the applications.

This paper will provide an opportunity to find out the necessity of development of sample lesson with Maple for learners who take Differential requirements in the Department of Science and Engineering rather than getting the simple algorism.

II. Theological background

2.1 The Computer and Differentiation

As computers have developed, they have contributed to the calculation of complex problems, quickly and accurately managing a lot of data. This development has created new way of calculating called the Numerical Approach, which has changed mathematical activities so that they are more experimental.

The reasons for introducing computer programming into math education are following;

First, error revise activities can improve mathematical thinking skill. Second, it could take a programming role in new curriculum. Thus, having the ability to solve the problems with computer programming is essential in order to be prepared for

an automatic and advanced society in the future. The ability of computers to experiment with mathematical objects and concepts as well as guesstimating and synthesizing possible results, provides probable proof of this theory. According to the distribution of the computer and computer science, universities have been trying to introduce the calculator and computer in their field; which is a common trend in the world.

Recently, the differentiation textbooks, which are used in many countries, include analytical geometry and have also been written so that they may be used with a computer software accompaniment to study the contents. Utilizing computer software in Mathematics teaching and learning means that problems are much more differentiated than before. Thus, with symbols, calculations get minimized, easily factored algebraic equations are used, and also 15° , 30° , 45° of 15 multiple numbers are only mentioned in angle related problems, which is unnecessary. The latest differentiation textbooks introduce Newton and Picard's method and practice the numerical application to attract computers in the field of Differential and Integral Calculus.

2.2 Educational Consideration of Computer Usage

Using a computer is considered a means to help teachers in class. There are a variety of ways to use the computer in lessons. Enlarged monitor screens can be used as a board with supporting equipment that is able to trace the image. An instructor can operate the computer and give examples, showing a picture. Individuals or groups can use personal

computers for programming, also they can use a packet. Several computers can be connected within a network. Furthermore, computers can be used for "practical work" in the computer labs. The common way to use a computer in academic math is that students are required to pass the course with questions and tests in a form of "tutoring", or CAI (Computer Aided Instruction). There has been equipment developed to assist mathematics teaching such as, software that does a sample experiment, a spread sheet that is used for mathematic purpose, and Micro world that users can control the objects. This equipment is used for learning throughout programming and represents the graphic packages and mathematical concepts. Computers can take several roles, like a tutor in CAI, a tutee who is the user of language, as programming, or as a tool, depending on the relation of learners and the level of usage (Taylor, 1980). A lot of software has been used as an assisting tool for learning and searching lately. Controlling the learning environment, active learning is encouraged and learners begin to think and figure out things by themselves.

Is the computer use always beneficial in learning? This should be kept in tutors and software producers' mind. For example, graphics software can draw many geometric shapes in a little amount of time with accuracy. However, it is different matter with helping to learn. Software does not guarantee improvement in study ability. Teaching strategy plays an important role in that. The ultimate matter is whether the use of the computer affects the learners' knowledge, how the knowledge is structured, and the way that the learner utilizes it.

2.3 The Function and Feature of Maple

Maple is a language to deal with the mathematics problems, which is a symbolized calculation system or a computational algebraic system.

K. Geddes and G. Gonnet in Waterloo University, Canada developed an epoch-making program that could algebraically manage the numerical formulas or algebraic equations in the 1980s. Starting from Maple V, the program has been developed until Maple v16.

Most math programs usually needed to input the figures with all the variables but Maple includes a basic function that can calculate unsupplied symbols. Also, equations with symbols can be simplified and converted in Maple. Among the advantages of Maple, numerical formulas provide ease in defining and expressing a function. Also accurate figures are obtainable Symbolic computation, graphic and animation operations are available as well.

Maple's features are as follows:

- It is easy to learn because numerical formulas in math are applied to the program.
- Most functions in math are definable and it is easy to represent them.
- Unlike the value from the numeric computation, it is possible to get accurate result.
- It is possible to do symbolic computation.
- It deals with variable graphics quickly and it includes plentiful options to embody many graphics.
- It realizes animations.
- It is an advanced level of programming language with diverse built-in functions.
- It is easily written.
- It helps with understanding in mathematical

concepts when used in differential and integral calculus, the number theory, a differential equation, linear algebra, and statistics.

- Features of Imperative Sentences

- ① The sentence ends with a semicolon (;).
- ② It includes parenthesis () after the sentence.
- ③ The equation is defined with an imperative word (:=).
- ④ Symbol (->) is used to appoint a function.
- ⑤ The calculation of accurate equation and approximate value is classified.

2.4 Maple and Mathematics

Maple is structured with a basic program, Kernel, categorized programs in a package form. Currently, categories in a packaged file in a Window program (version 16) for PC in IBM are algebra, number theory, graphic, statistics, numerical analysis, and differential and integral calculus.

Maple includes all differential related functions. It is utilized in the slope of the tangent, the differential and graph of polynomial functions, rational functions, trigonometric functions, the partial and definite differential of multi-variate functions, as well as partial differential and definite integral, indefinite integral, double integral, the medial theorem, a limit, 3D graphs, and curves of two-variate function $z = f(x, y)$. Differential (D()) or 2D and 3D graphs (plot()), plot3d()), which are commonly used contents, are included in the basic program as a function to practice easily.

Differential of a implicative function (implicitdiff()) and animating skill (animate()), which are relatively long in frequency, are only practiced with

need, as they are categorized in differential and integral calculus, algebra, discrete mathematics and geometry.

The difference between this and the similar program Mathematica is that the first letter of imperative words does not need to be a capital letter.

III. Differential Study Practice through Maple

3.1 Precautions in Maple Activity

As it is mentioned in chapter II, Maple can quickly represent the accurate graph with formulated functions or numeric equations. It can get the result of complex matrix calculation and procedures of differential and integral result. Even if getting the result is easily done utilizing the built-in functions in a program, it does not mean that the learner understands the concept or principle of differential and it is not necessarily beneficial to learning.

If it is required to get the simple answer from calculations in engineering it might be advantageous, however, in a view of understanding of differential and integral concept, Maple's practice is not necessarily required. Thus, to efficiently use Maple in differential and integral study the following basic rules should be established.

① It should be used to help understand the mathematical concept, explaining the difficulty of use in a paper-base environment since it has outstanding calculating functions, and variable expressing methods.

② The variable functions represented in a 3-dimensional space should be visualized and they

should be used to extend the range of the mathematical thinking and problem solving skill with utilizing advantages, expanding the concept of graph on the plane.

③ Throughout (this process of) solving complex situations, which were not dealt with in the past paper-based environment, the differential and integral application ability should be distributed to have positive recognition in practical use.

3.2 Differential Practice with Maple

(1) The slope of the tangent

Generally, there is a relation between the concept of differential with the slope of the tangent and the result of differential with Maple's imperative word 'diff'. The slope of the tangent at $x = \alpha$ resulted from limit value the slope of the secant of to the two points $(\alpha, f(\alpha))$ and $(\beta, f(\beta))$. The following is the slope of the secant do two points using maple.

```
> with(student):
> a := slope([beta, f(beta)], [alpha, f(alpha)]);
```

$$a := \frac{f(\beta) - f(\alpha)}{\beta - \alpha}$$

그림 1. 활선의 기울기
Fig. 1. The Slope of the Secant

Let us get the slope of the tangent on the curve $y = (x - 1)^2 + 5$ at the point $(2, 6)$. First, let be $\alpha = 2$ and get the sequence of point $\beta = \alpha + h$ when h approximates at 0 infinitely. (<Fig. 2>)

When we go h close to 0 infinitely the accurate slope of the tangent is obtained by getting the limit value of the slopes, according to the <Figure 2>.

```

> f := x -> (x-1)^2 + 5;
> alpha := 2; beta := alpha + h;
> h := 0.5 : slope([beta, f(beta)], [alpha, f(alpha)]);
> h := 0.1 : slope([beta, f(beta)], [alpha, f(alpha)]);
> h := 0.05 : slope([beta, f(beta)], [alpha, f(alpha)]);
> h := 0.01 : slope([beta, f(beta)], [alpha, f(alpha)]);
> h := 0.005 : slope([beta, f(beta)], [alpha, f(alpha)]);
> h := 0.001 : slope([beta, f(beta)], [alpha, f(alpha)]);
> h := 0.0005 : slope([beta, f(beta)], [alpha, f(alpha)]);

```

$f := x \rightarrow (x-1)^2 + 5$
$\beta := 2 + h$
2.500000000
2.100000000
2.050000000
2.010000000
2.005000000
2.001000000
2.000500000

그림 2. β 의 수열들
Fig. 2. The sequences of β

```

> h := 'h';
> Limit(slope([beta, f(beta)], [alpha, f(alpha)], h=0);
> a := limit(slope([beta, f(beta)], [alpha, f(alpha)], h=0);

```

$\lim_{h \rightarrow 0} \frac{(1+h)^2 - 1}{h}$
$a := 2$

그림 3. 그림 2에서 구하였던 기울기들의 극한값
Fig. 3. the limit of the slopes, according to the Fig. 2.

Point (2,6) and the curve $f(x)$ are represented with Maple in the following:

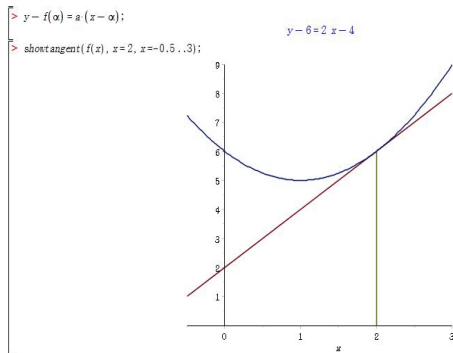


그림 4. 함수 $y = (x-1)^2 + 5$ 의 그래프와 점 (2,6)에서의 접선
Fig. 4. The graphs of function $y = (x-1)^2 + 5$ and tangent line at pt. (2,6)

(2) Chain Rule

A derivative of the function $y = \sin(x^2 + x - 5)$ is assigned. In general differential, the derivative of trigonometric function $y = \sin x$ is easily calculated. The previously given function, needs to be calculated with differential of chain rule with this format.

$$\begin{cases} y = \sin t \\ t = x^2 + x - 5 \end{cases}$$

Maple results.

```

> f := x -> sin(x);
> g := x -> x^2 + x - 5;
> h := f@g;
> h(x);

```

$\sin(x^2 + x - 5)$

그림 5. Maple 결과
Fig. 5. Maple results

(3) Higher derivatives

$y' = \cos x$ is the first derivative of trigonometric function $y = \sin x$. y' is also a function for x . So we can derivative the function $y' = \cos x$ for x . It is written by $y'' = -\sin x$. Similarly, we'll express with $y''' = y^{(3)} = -\cos x$, $y'''' = y^{(4)} = \sin x$.

According to Maple,

```

> f := sin(x);
> f1 := diff(f, x);
> f2 := diff(f, x, x);
> f3 := diff(f, x, x, x);
> f4 := diff(f, x, x, x, x);

```

$f := \sin(x)$
$f1 := \cos(x)$
$f2 := -\sin(x)$
$f3 := -\cos(x)$
$f4 := \sin(x)$

그림 6. 고계도함수 Maple 결과
Fig. 6. Maple results of Higher derivatives

(4) The derivative of function with several variables

Maple proves that the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2+y^2}$ of two-variable function do not existed.

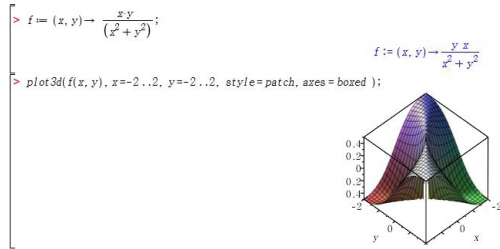


그림 7. 함수 $z = f(x,y) = \frac{xy}{x^2+y^2}$ 의 그래프

Fig. 7. The graph of function $z = f(x,y) = \frac{xy}{x^2+y^2}$.

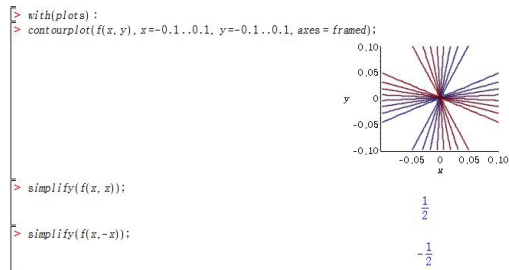


그림 8. 함수 $z = f(x,y) = \frac{xy}{x^2+y^2}$ 의 등고선 그래프

Fig. 8. The contour graph of function $z = f(x,y) = \frac{xy}{x^2+y^2}$.

Fig. 8. shows that the function $f(x)$ shifts characteristically near $(0,0)$. Also along the line $y = x$ is resulted $f(x, y) = f(x, x) = \frac{1}{2}$, but $f(x, -x) = -\frac{1}{2}$ is represented along the line $y = -x$. Therefore, when (x, y) gets infinitely close to $(0,0)$, the function $f(x, y)$ does not have the limit value.

IV, Summary and Conclusion

In the 21st Century, a knowledge-based age, computers should be utilized as an supporting equipment for mathematics courses. Computers have made it possible to manage difficult problems, including numeric analysis, which was not available in the past.

Maple is a suitable software program for differentiation lectures in university, because it provides variable services like graphic function, numeric calculation, algebraic computation and programming. Maple can be demonstrated to practice concepts and principles. It has an outstanding visualizing function, in addition to calculations, so it affects mathematics causing a change from deductive to significant math. In addition, to getting the algorithm through the program, problem solving skills get close to the life and mathematics' practical use is being contributed.

Maple is not a simple calculator, but a program to learn math. Therefore, the instructor's role is very important when using Maple in a lecture. Instructors should spend a lot of time studying Maple's direction as well as researching.

In conclusion, programs should be written and all data need to be collected in order to be beneficial in all areas.

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