



Interval Distance for Similarity Measure Between Interval-valued Fuzzy Numbers

Byoung-Chan Jeon¹, Sang Yeop Cho²

¹*Division of Liberal Arts, Chungwoon University*

²*Department of Internet, Chungwoon University*

ABSTRACT

Fuzzy systems such as fuzzy linear systems, fuzzy differential equations, fuzzy linear programming, fuzzy decision making, and other topics are used to study various problems. In general, fuzzy numbers are used to represent the various data in fuzzy systems. Many researches have worked on that measure the distance or the different between two fuzzy numbers as follows: computing distance between two generalized fuzzy numbers, distance measuring between interval numbers, measuring between fuzzy numbers in the Euclidean space, measuring include vagueness, distance measuring between Gaussian type fuzzy numbers, measuring interval metric distance include α -cut, and so on. Distance measure or different are very essential tool in a variety of fields. In this paper, we propose a method to measure the interval distance of two interval-valued fuzzy numbers. The method is based on the measure of interval distance on the research of M. A. Firozja which can apply to two interval fuzzy numbers. Also, we show the properties of interval distance and proofs of them. The interval distance between interval-valued fuzzy numbers can provide us with a useful way to measure the similarity for handling various fuzzy systems which represent the data as interval-valued fuzzy number.

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KEYWORDS : Fuzzy distances, Interval distances, Interval-valued fuzzy numbers, Interval-valued fuzzy sets, Level sets

ARTICLE INFO: Received 29 July 2014, Revised 18 August 2014, Accepted 18 August 2014.

1. Introduction

*Corresponding author is with the Department of Internet, Chungwoon University, 25 Daehak-gil Hongseong-eup Hongseong-gun Chungnam, 350-701, KOREA.
E-mail addresses: sycho@chungwoon.ac.kr

There are several researches to evaluate the distance between fuzzy numbers. The distance

measure evaluate the distance or difference between two fuzzy numbers and is a very useful method in various fields. Many researchers have studied on distance measures between fuzzy numbers[1-9]. In [1] Tran et al. suggested the distance measure between the interval numbers. In [2] Diamond et al. proposed a measure between fuzzy numbers in the Euclidean space. In [3] Yang et al. updated the measure proposed by Diamond with measure that is able to include more information about vagueness. In [4] Xu et al. introduced the distance measure between Gaussian type fuzzy numbers. In [5] Cheng suggested a distance index using the centroid points of the fuzzy numbers. In [6] Allahviranllo et al. proposed the distance for two fuzzy numbers on the interval numbers. In [7] Chakraborty et al. compute the fuzzy distance for two generalized fuzzy numbers. In [8] Guha et al. introduced the fuzzy distance between two generalized fuzzy numbers and confidence level of fuzzy numbers is considered. In [9] Firozja et al. studied the distance for two fuzzy numbers by their α -cut, an interval metric distance on fuzzy numbers.

In this study, we propose the distance measure between interval-valued fuzzy numbers based on the measure suggested by Firozja et al.[9] who describe the interval distance between generalized fuzzy numbers.

The paper is organized as follows. In Section 2, we briefly state some basic concepts. Section 3 describes interval distance and Section 3 proves the properties of the metric for distance measure. finally, the paper ends with conclusions.

2. Background concepts

In this section, some basic concepts of the generalized left right fuzzy number are given and then some concepts of interval-valued fuzzy number are described.

2.1 generalized left right fuzzy number

To be able to deal with curvilinear membership functions, we define the generalized left right fuzzy number(GLRFN) as the following definition 1[10].

Definition 1. $\tilde{A}_{GLRFN}=(a_1, a_2, a_3, a_4)_{GLRFN}$ is a GLRFN whose membership function satisfies the following:

$$\mu_{\tilde{A}_{GLRFN}}(x) = \begin{cases} L(\frac{a_2-x}{a_2-a_1}), & a_1 \leq x \leq a_2, \\ 1, & a_2 \leq x \leq a_3, \\ R(\frac{x-a_3}{a_4-a_3}), & a_3 \leq x \leq a_4, \\ 0, & otherwise. \end{cases} \quad (1)$$

where L and R are strictly decreasing functions defined on [0,1] and satisfying the conditions:

$$\begin{aligned} L(t) = R(t) = 1 & \text{ if } t \leq 0, \\ L(t) = R(t) = 0 & \text{ if } t \geq 1. \end{aligned}$$

For $a_2=a_3$, we have the classical definition of left right fuzzy numbers(LRFN). Trapezoidal fuzzy numbers(TrFN) are special cases of GLRFN with $L(t) = R(t) = 1 - t$ as the following definition 2.

Definition 2. $\tilde{A}_{TrFN}=(a_1, a_2, a_3, a_4)_{TrFN}$ is a TrFN whose membership function satisfies the following:

$$\mu_{\tilde{A}_{TrFN}} = \begin{cases} \frac{a_2 - x}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ 1, & a_2 \leq x \leq a_3, \\ \frac{x - a_3}{a_4 - a_3}, & a_3 \leq x \leq a_4, \\ 0, & otherwise. \end{cases} \quad (2)$$

Triangular fuzzy numbers(TFN) are also special cases of GLRFN with $L(t) = R(t) = 1 - t$ and $a_2=a_3$. r-level interval of fuzzy number \tilde{A} as $[\tilde{A}]^r = \{x \in \mathbb{R} \mid \mu_{\tilde{A}}(x) \geq r\}$ where $[\tilde{A}]^r = [\underline{A}(r), \overline{A}(r)] = [a_2 - (a_2 - a_1)L_r^{-1}(r), a_3 + (a_4 - a_3)R_r^{-1}(r)]$.

Let $F(\mathbb{R})$ be the fuzzy numbers on \mathbb{R} and $I(\mathbb{R})$ be the interval numbers on \mathbb{R} , then we denote $[a, b]$ as a interval number where $a \leq b$. Let the two interval numbers $[a, b], [c, d] \in \mathbb{R}$ and $\tau \in \mathbb{R}$, then we can define the operations of the interval numbers as followings:

1. $[a, b] \oplus [c, d] = [a+c, b+d]$,
2. $\tau \odot [a, b] = [\min(\tau a, \tau b), \max(\tau a, \tau b)]$.

The comparison for two interval numbers $[a, b]$, and $[c, d]$ is the followings:

1. $[a, b] \leq [c, d] \equiv a \leq c, b \leq d$,
2. $[a, b] = [c, d] \equiv a = c, b = d$,
3. $[a, b]$ is nonnegative such that $[a, b] \geq [0, 0]$, and write $[a, b] \geq 0$, and $a \leq b \equiv \min(a, b) = a$.

2.2 interval-valued fuzzy number

We briefly describe the trapezoidal interval-valued fuzzy number(TrIvFN)[11].

Definition 3. $\tilde{A} = \langle \tilde{A}^L, \tilde{A}^U \rangle = \langle (a_1^L, a_2^L, a_3^L, a_4^L; \mu_{\tilde{A}^L}(x)), (a_1^U, a_2^U, a_3^U, a_4^U; \mu_{\tilde{A}^U}(x)) \rangle$ is a TrIvFN whose membership function satisfies the following:

$$\mu_{\tilde{A}^L}(x) = \begin{cases} \frac{a_2^L - x}{a_2^L - a_1^L}, & a_1^L \leq x \leq a_2^L, \\ \omega^L, & a_2^L \leq x \leq a_3^L, \\ \frac{x - a_3^L}{a_4^L - a_3^L}, & a_3^L \leq x \leq a_4^L, \\ 0, & otherwise. \end{cases} \quad (3)$$

$$\mu_{\tilde{A}^U}(x) = \begin{cases} \frac{a_2^U - x}{a_2^U - a_1^U}, & a_1^U \leq x \leq a_2^U, \\ \omega^U, & a_2^U \leq x \leq a_3^U, \\ \frac{x - a_3^U}{a_4^U - a_3^U}, & a_3^U \leq x \leq a_4^U, \\ 0, & otherwise. \end{cases} \quad (4)$$

where TrIvFN \tilde{A} has two elements, where the one is the lower trapezoidal interval-valued fuzzy number $\mu_{\tilde{A}^L}(x)$, and the other one is the upper trapezoidal interval-valued fuzzy number $\mu_{\tilde{A}^U}(x)$, and $a_1^L \leq a_2^L \leq a_3^L \leq a_4^L, a_1^U \leq a_2^U \leq a_3^U \leq a_4^U, a_1^L \leq a_1^U \leq a_2^L \leq a_2^U \leq a_3^L \leq a_3^U \leq a_4^L \leq a_4^U, 0 \leq \omega^L \leq \omega^U \leq 1$.

3. Interval distance

In this section, we present the interval distance between interval-valued fuzzy numbers based on the interval distance between generalized fuzzy numbers.

Let \tilde{A} and \tilde{B} be TrIvFN with $\tilde{A} = \langle \underline{A}^L, \bar{A}^U \rangle$ and $\tilde{B} = \langle \underline{B}^L, \bar{B}^U \rangle$ and then also with $[\tilde{A}^L]^r = [\underline{A}^L(r), \bar{A}^L(r)]$, $[\tilde{A}^U]^r = [\underline{A}^U(r), \bar{A}^U(r)]$, $[\tilde{B}^L]^r = [\underline{B}^L(r), \bar{B}^L(r)]$, $[\tilde{B}^U]^r = [\underline{B}^U(r), \bar{B}^U(r)]$ respectively.

We can evaluate the distance between these two interval-valued fuzzy numbers as defined $d(\tilde{A}, \tilde{B}) = [(\underline{d}^L + \underline{d}^U)/2, (\bar{d}^L + \bar{d}^U)/2]$, where

$$\underline{d}^L = \min \{ \min_{r \in [0,1]} | \underline{A}^L(r) - \underline{B}^L(r) |, \min_{r \in [0,1]} | \bar{A}^L(r) - \bar{B}^L(r) | \},$$

$$\bar{d}^L = \max \{ \max_{r \in [0,1]} | \bar{A}^L(r) - \bar{B}^L(r) |, \max_{r \in [0,1]} | \underline{A}^L(r) - \underline{B}^L(r) | \},$$

$$\underline{d}^U = \min \{ \min_{r \in [0,1]} | \underline{A}^U(r) - \underline{B}^U(r) |, \min_{r \in [0,1]} | \bar{A}^U(r) - \bar{B}^U(r) | \},$$

$$\bar{d}^U = \max \{ \max_{r \in [0,1]} | \bar{A}^U(r) - \bar{B}^U(r) |, \max_{r \in [0,1]} | \underline{A}^U(r) - \underline{B}^U(r) | \}.$$

4. Properties

The proposed distance measure between interval-valued fuzzy numbers has the following properties, where the proofs of these properties are shown.

Property 1. Two interval-valued fuzzy numbers \tilde{A} and \tilde{B} are identical iff $d(\tilde{A}, \tilde{B}) = [0, 0]$.

Proof. If two interval-valued fuzzy numbers \tilde{A} and \tilde{B} are identical, then $\underline{A}^L(r) = \underline{B}^L(r)$, $\bar{A}^L(r) = \bar{B}^L(r)$, $\underline{A}^U(r) = \underline{B}^U(r)$, $\bar{A}^U(r) = \bar{B}^U(r)$. we can get the interval values of the lower fuzzy number and the upper fuzzy number as follows: $\underline{d}^L = \min \{ \min_{r \in [0,1]} | \underline{A}^L(r) - \underline{B}^L(r) |, \min_{r \in [0,1]} | \bar{A}^L(r) - \bar{B}^L(r) | \}$

$$= 0, \quad \bar{d}^L = \max \{ \max_{r \in [0,1]} | \bar{A}^L(r) - \bar{B}^L(r) |, \max_{r \in [0,1]} | \underline{A}^L(r) - \underline{B}^L(r) | \} = 0, \quad \underline{d}^U = \min \{ \min_{r \in [0,1]} | \underline{A}^U(r) - \underline{B}^U(r) |, \min_{r \in [0,1]} | \bar{A}^U(r) - \bar{B}^U(r) | \} = 0, \quad \text{and} \quad \bar{d}^U = \max \{ \max_{r \in [0,1]} | \underline{A}^U(r) - \underline{B}^U(r) |, \max_{r \in [0,1]} | \bar{A}^U(r) - \bar{B}^U(r) | \} = 0.$$

Therefore, we can obtain

$$\begin{aligned} d(\tilde{A}, \tilde{B}) &= [(\underline{d}^L + \underline{d}^U) / 2, (\bar{d}^L + \bar{d}^U) / 2] \\ &= [(0 + 0) / 2, (0 + 0) / 2] \\ &= [0, 0]. \end{aligned}$$

Property 2. $d(\tilde{A}, \tilde{B}) = d(\tilde{B}, \tilde{A})$.

Proof. we can see that $d(\tilde{A}, \tilde{B}) = [(\underline{d}^L + \underline{d}^U)/2, (\bar{d}^L + \bar{d}^U)/2]$, where $\underline{d}^L = \min \{ \min_{r \in [0,1]} | \underline{A}^L(r) - \underline{B}^L(r) |, \min_{r \in [0,1]} | \bar{A}^L(r) - \bar{B}^L(r) | \}$, $\bar{d}^L = \max \{ \max_{r \in [0,1]} | \bar{A}^L(r) - \bar{B}^L(r) |, \max_{r \in [0,1]} | \underline{A}^L(r) - \underline{B}^L(r) | \}$, $\underline{d}^U = \min \{ \min_{r \in [0,1]} | \underline{A}^U(r) - \underline{B}^U(r) |, \min_{r \in [0,1]} | \bar{A}^U(r) - \bar{B}^U(r) | \}$, $\bar{d}^U = \max \{ \max_{r \in [0,1]} | \bar{A}^U(r) - \bar{B}^U(r) |, \max_{r \in [0,1]} | \underline{A}^U(r) - \underline{B}^U(r) | \}$.

$$\begin{aligned} d(\tilde{B}, \tilde{A}) &= [(\underline{d}^L + \underline{d}^U)/2, (\bar{d}^L + \bar{d}^U)/2], \text{ where } \underline{d}^L \\ &= \min \{ \min_{r \in [0,1]} | \underline{B}^L(r) - \underline{A}^L(r) |, \min_{r \in [0,1]} | \bar{B}^L(r) - \bar{A}^L(r) | \}, \\ \bar{d}^L &= \max \{ \max_{r \in [0,1]} | \bar{B}^L(r) - \bar{A}^L(r) |, \max_{r \in [0,1]} | \underline{B}^L(r) - \underline{A}^L(r) | \}, \\ \underline{d}^U &= \min \{ \min_{r \in [0,1]} | \underline{B}^U(r) - \underline{A}^U(r) |, \min_{r \in [0,1]} | \bar{B}^U(r) - \bar{A}^U(r) | \}, \\ \bar{d}^U &= \max \{ \max_{r \in [0,1]} | \bar{B}^U(r) - \bar{A}^U(r) |, \max_{r \in [0,1]} | \underline{B}^U(r) - \underline{A}^U(r) | \}, \\ \text{where } \min_{r \in [0,1]} | \underline{A}^L(r) - \underline{B}^L(r) | &= \min_{r \in [0,1]} | \underline{B}^L(r) - \underline{A}^L(r) |, \quad \min_{r \in [0,1]} | \bar{A}^L(r) - \bar{B}^L(r) | \\ &= \min_{r \in [0,1]} | \bar{B}^L(r) - \bar{A}^L(r) |, \quad \min_{r \in [0,1]} | \underline{A}^U(r) - \underline{B}^U(r) | \\ &= \min_{r \in [0,1]} | \underline{B}^U(r) - \underline{A}^U(r) |, \quad \text{and} \quad \min_{r \in [0,1]} | \bar{A}^U(r) - \bar{B}^U(r) | \\ &= \min_{r \in [0,1]} | \bar{B}^U(r) - \bar{A}^U(r) |. \end{aligned}$$

Therefore, we can obtain $d(\tilde{A}, \tilde{B}) = d(\tilde{B}, \tilde{A})$.

Property 3. If \tilde{A} and \tilde{B} are real numbers then $d(\tilde{A}, \tilde{B}) = |\mathbf{a-b}|$.

Proof. If \tilde{A} and \tilde{B} are two real numbers, then we can see that

$$\begin{aligned} \tilde{A} &= \langle \tilde{A}^L, \tilde{A}^U \rangle \\ &= \langle (a_1^L, a_2^L, a_3^L, a_4^L; \mu_{\tilde{A}}(x)), (a_1^U, a_2^U, a_3^U, a_4^U; \mu_{\tilde{A}}(x)) \rangle \\ &= \langle (\mathbf{a}, \mathbf{a}, \mathbf{a}, \mathbf{a}; 1), (\mathbf{a}, \mathbf{a}, \mathbf{a}, \mathbf{a}; 1) \rangle \\ &= (\mathbf{a}, \mathbf{a}, \mathbf{a}, \mathbf{a}; 1) \\ &= \mathbf{a}, \\ \tilde{B} &= \langle \tilde{B}^L, \tilde{B}^U \rangle \\ &= \langle (b_1^L, b_2^L, b_3^L, b_4^L; \mu_{\tilde{B}}(x)), (b_1^U, b_2^U, b_3^U, b_4^U; \mu_{\tilde{B}}(x)) \rangle \\ &= \langle (\mathbf{b}, \mathbf{b}, \mathbf{b}, \mathbf{b}; 1), (\mathbf{b}, \mathbf{b}, \mathbf{b}, \mathbf{b}; 1) \rangle \\ &= (\mathbf{b}, \mathbf{b}, \mathbf{b}, \mathbf{b}; 1) \\ &= \mathbf{b}, \end{aligned}$$

we can get $\min_{r \in [0,1]} |\underline{A}^L(r) - \underline{B}^L(r)| = |\mathbf{a-b}|$, $\min_{r \in [0,1]} |\overline{A}^L(r) - \overline{B}^L(r)| = |\mathbf{a-b}|$, $\min_{r \in [0,1]} |\underline{A}^U(r) - \underline{B}^U(r)| = |\mathbf{a-b}|$, and $\min_{r \in [0,1]} |\overline{A}^U(r) - \overline{B}^U(r)| = |\mathbf{a-b}|$, and also calculate as follows $\underline{d}^L = \min_{r \in [0,1]} |\underline{A}^L(r) - \underline{B}^L(r)|$, $|\min_{r \in [0,1]} |\overline{A}^L(r) - \overline{B}^L(r)|| = |\mathbf{a-b}|$, $\overline{d}^L = \max_{r \in [0,1]} |\overline{A}^L(r) - \overline{B}^L(r)| = |\mathbf{a-b}|$, $\underline{d}^U = \min_{r \in [0,1]} |\underline{A}^U(r) - \underline{B}^U(r)|$, $|\min_{r \in [0,1]} |\overline{A}^U(r) - \overline{B}^U(r)|| = |\mathbf{a-b}|$, and $\overline{d}^U = \max_{r \in [0,1]} |\overline{A}^U(r) - \overline{B}^U(r)| = |\mathbf{a-b}|$.

Therefore, we can obtain $d(\tilde{A}, \tilde{B}) = [|\mathbf{a-b}|, |\mathbf{a-b}|] = |\mathbf{a-b}|$.

5. Conclusions

In this paper, we have described a distance measure between interval-valued fuzzy numbers based on the measure suggested by Firozja et al. who describe the interval distance between

generalized fuzzy numbers. The proposed distance measure is more flexible than the other methods due to the fact that it uses interval-valued fuzzy numbers. The proposed method can apply to fuzzy similarity measure, fuzzy decision making, fuzzy risk analysis problems, and various fuzzy systems such as marin power plants.

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의 거리척도, 유클리드 공간에서의 퍼지 숫자의 척도, 모호함을 포함한 척도, 가우스 형 숫자의 척도 α -자르기를 포함한 구간 거리척도 등 많은 연구가 두 개의 퍼지숫자 사이의 거리 또는 차이를 척도를 하는 연구를 하고 있다. 거리척도 도는 차이는 다양한 분야에서 기본적인 도구로 사용한다. 본 논문에서는 구간값 퍼지 숫자에 적용할 수 있는 구간 거리를 척도하는 방법을 제안한다. 이 방법의 두 개의 구간 퍼지 숫자에 적용할 수 있는 M. A. Firozjar 제안한 구간 거리의 척도에 기반을 두고 있다. 또한 우리는 구간거리의 속성과 이들의 증명도 제시한다. 구간값 퍼지 숫자의 구간거리는 구간값 퍼지 숫자로 자료를 표현하는 다양한 퍼지 시스템을 다루기 위한 유사도를 척도하는 유용한 방법을 제공할 수 있다.

Acknowledgments

This work was supported by the Research Fund of Chungwoon University in 2013.



Byoung-Chan Jeon received the bachelor's degree in the department of computer science from the Hanbat National University in 1989. He received the M.S. degree in the department of computer science from Suwon University in 1994. He received the Ph.D. degree in the department of computer science and engineering from Soonchunhyang University in 2001. He is currently a professor in the Department of Liberal Arts at Chungwoon University, Korea. His current research interests include computer architecture, network application, embedded systems. He is a member of the KKITS.

E-mail : jbc66@chungwoon.ac.kr

구간값 퍼지숫자의 유사척도를 위한 구간 거리

전병찬¹, 조상엽²,

¹청운대학교 교양학부

²청운대학교 인터넷학과

요 약

퍼지선형 시스템, 퍼지 미분 방정식, 퍼지선형 프로그래밍, 퍼지의사 결정 등과 같은 퍼지 시스템은 다양한 문제를 연구하는데 사용한다. 일반적으로 퍼지숫자는 퍼지시스템의 다양한 데이터를 퍼지 숫자로 표현한다. 일반화된 퍼지 숫자의 거리 계산, 구간 숫자



Sang Yeop Cho received the bachelor's degree in the department of computer engineering from the Hannam University in 1986. He received the M.S. degree and the Ph.D.

degree in the Department of Computer Engineering from Chungang University in 1988 and 1993, respectively. He is currently a professor in the Department of Internet at Chungwoon University, Korea. His current research interests include artificial intelligence, intelligent systems, fuzzy sets. He is a life member of the KKITS,

E-mail : sycho@chungwoon.ac.kr