

Comparative Analysis of the Optimum Firm Location Under Profit Maximizing and Expense Preference Approach¹⁾

기업의 최적 입지 비교 분석
-수익 극대화 및 지출선호 모형을 중심으로 -

Jichung Yang

(Fellow)

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I . Introduction

The locations of production activities are not predetermined but are subject to economic choice. In general, availability of resources, location of population as a source of labour and as potential markets, soil, climate and technical conditions rule out many locations for any particular economic activity. What remains is a set of feasible locations among which an economic choice is to be made. In neoclassical economic theory these choices are seen as attempts to maximize profits(Beckmann 1985). In relation with the location choice, most of the papers deal with optimum location problem only for an unregulated firm in conjunction with a heterogeneous space where markets are given at discrete points (Mathurss 1979, Fujita 1981). Mai(1985) considered the optimum location of regulated firm with rate of return and Yang(1993) considered the case of regulated firm under agglomeration economies/diseconomies and congestion.

In this paper we consider the existence of congestion as well as agglomeration in the economy. The spatial settings are same as our previous work(Yang 1993) with linear location line. Capital is ubiquitous and labor is concentrated at the single site. However, Yang(1993) only introduced the optimum location of profit maximizing approach even though there are various approaches such as firm value maximizing, expense preference approach and so on in the point of firm and industry analysis. Recently the modern firm is experiencing the organizational transformation related to managerial specialization and the consequent separation of the firm's management from its ownership. Hence there is a possibility of some gaps between manager's decisions and the owner's profit interests. In this paper, we focus on the locational comparison between profit maximizing approach and expense preference model approach after reviewing the analytical framework including regulated firm's optimum location, well-known overcapitalization effects, and agglomeration economies.

Our first work was to derive the regulated firm's optimum location under

agglomeration economies. However, for these processes we used the profit maximizing approach.

The starting point for this paper is that we do not know the difference between the location of profit maximizing approach and expense preference model approach. What is the main parameters deciding optimum location from two different approaches. Hence it is very meaningful to compare the optimum location of two different approaches. The industrial location and related policies including location subsidies should move towards elevating the efficiency reviewing various industrial transformation and modern firm micro-behaviour.

II. Theoretical Background and Model Development

According to Averch and Johnson(1962), the introduction of an active constraint of a fair rate of return would induce the firm to invest more than the original profit maximizing value of capital, and this would create an inefficient allocation of inputs. Hence, the Averch-Johnson(A-J) model suggests that because of rate-of return regulation, the regulated firm has an incentive to overcapitalize in its production. This is known as the "overcapitalization" effect or the A-J bias. However, all of these discussions for examining A-J effects were implemented within the context of a spaceless economy, even though real economy is characterized by dispersion of customers and producers over geographic space, with trade between them always incurring transport costs. Moreover, it is our common belief that there exist an agglomeration economy/diseconomy and congestion (Yang, 1993). Therefore we introduced this environment to analytical settings. In our previous work, we considered a big company town(city) producing a homogeneous output(Q) using capital (K), labor (L) and one material(M). The aggregate production function of the industrial city (firm) was:

$$Q = f(K, L, M) \tag{1a}$$

where the first and second order derivatives, $f_K > 0$ $f_L > 0$ $f_M > 0$ $f_{KK} < 0$ $f_{LL} < 0$ $f_{MM} < 0$. For simplicity, we considered the firm's locational decision where the market for the firm's final products concentrated at the single site, C, city. This is a problem of company town location decision.

Hence, the theoretical framework is composed of only one big firm producing homogeneous output(Q) using capital(K) labor(L), and one material(M). For example, this is the case of automobile, petroleum refinery, etc.

In order to make comparison the optimum location under profit maximizing approach with the optimum location under expense preference approach, we need to review previous work in detail.

Assume capital and labor are ubiquitous and can thus be obtained anywhere on the straight line, MC, between M and C, firm uses M as a intermediate input which is located only at M site. Define y as the distance from M to C, city, and x as the distance from C to the firm's location at F and restrict $0 < x < y$. For simplicity $y=1$. Define the inverse demand function as

$$P=P(Q) \tag{1b}$$

where $P'(Q) < 0$.

Assume that the firm has no control over the base price of input.

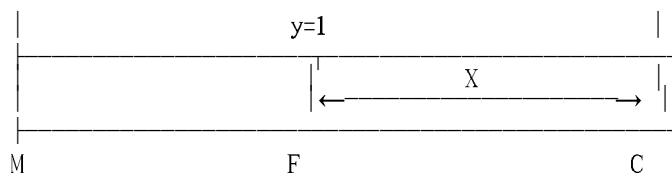


Figure 1. Location Line

We assumed that the production is affected by other factors besides this neoclassical production function such as the agglomeration economies/diseconomies and congestion effects. Firstly, in our previous work only agglomeration factors

were introduced under the regulatory constraint. Hence the aggregate production function under the agglomeration(scale) economies/diseconomies becomes

$$Y = A(N)f(N, K, M) \quad (2)$$

Y : aggregate output, N : aggregate labor

where A(N) is a scale economy/diseconomy(broadly speaking, agglomeration economy/diseconomy) shift factor and Y is aggregate output²⁾

$$\text{For simplicity, } A(N)=N^\alpha \quad \alpha \neq 0 \quad (3)$$

(if $\alpha > 0$ it means economy. if $\alpha < 0$ it means diseconomy)

The first rigorous analysis of a supply oriented urban model with agglomeration economies is offered by Rabenau(1979).

Production function $Q=A(N)K^a N^{1-a}$

$A(N)=N^b$ where $0 < a < 1$, $0 < 1-a+b < 1$ and b is positive or negative, depending on whether there are agglomeration economies or diseconomies in urban production activities.

Hanson(1979) et al introduced a public capital into urban growth model. Dendrinos(1982) assumes agglomeration economies as well as congestion. Miyao(1987) used similar production for urban growth model.

$$f(N, K, M)=N^a K^b M^{1-a-b} \quad a,b > 0 \quad (4)$$

The profit of the firm now can be described as

$$= [P(Y)-k(x)x] A(N)f(N,K,M)-M \cdot (y-x)t(x)-pM-wN-rK \quad (5)$$

where

$k(x)$ =transport rate on the firm's product

2) We used Q as homogeneous output and Y as aggregate output.

$t(x)$ =transport rate on the firm's intermediate input

w =price of a unit of labor

r =cost of a unit of capital

p =price of intermediate input, M

Following Averch and Johnson(1962), the regulated firm is permitted to earn no more than some fixed portion of the value of its capital—the regulatory fair rate of return—on its rate base.

Hence, the regulatory constraint can be written as

$$\{ [P(Y)-k(x)x] Y-M [(y-x)t(x)+P] -wN \} /k \leq g \quad (6)$$

where g is defined as the fair rate of return.

If g is less than r , profits are negative as the firm will refer to shut down. If $g=r$, the firm is indifferent between shutting down and operating(Mai 1985).

Following the literature, we will assume that the fair rate of return exceeds the cost of capital, Hence we have

$$g > r$$

Next, the firm is assumed to select the values of N, K, M and x that maximize (5) subject to (2) and (6)³).

Also, in this spatial setting, we could find that intermediate location is only possible in long haul economy⁴). If we consider land price or rent as an

3) Please refer to the appendix for nonconstrained maximization problem.

4) Our problem is simply to minimize the distance dependent cost, $C(x)$ for the purpose of finding firm's optimum location of profit maximization.

$$\begin{aligned} C(x) &= T(M)+T(Q) \\ &= M [t(x)(1-x)] +Q [k(x)x] \\ &= M [t_1(1-x)^\sigma+Q [k_1x^\sigma] \end{aligned}$$

where $T(M)$, $T(Q)$ are transport cost of M , Q , respectively.

if $\sigma=1$ liner (case a)

$\sigma>1$ long haul diseconomy (case b)

$0<\sigma<1$ long haul economy (case c)

Under the regulatory constraint, the distance dependent cost term $c(x)$ becomes $(1+\mu)(T(M)+T(Q))$. In case of linear transport rate, switching point

input factor, owing to the difference in land price between city and undeveloped area of MC the intermediate location is also feasible. Hence, it is possible that there exists intermediate location if we replace labor input with land input. Rent, R_n will be introduced instead of wage, w . In space economy, firm produce output and decide location in order to maximize profit.

III. Firm Location Under Profit Maximizing Approach

We can assume that there exist congestion effects in transport systems as well as agglomeration economies/diseconomies. Between M and C, we can introduce congestion effects. Local government deals with the transportation system, and there exists local variation of public services. Hence, we may assume that public services including transportation systems such as highway width, lane, etc are given. The congestion in transport system especially in highways can be derived by using the relationships among mf (mean rate of flow), sm (space mean speed), and md (mean density).

Hence, we can say that certain highway is congested after md exceeds the md_m [the density at which the flow rate is a maximum($mf = mf_m$)]⁵⁾.

With the introduction of congestion, transport cost will be increased. Let us assume the level of congestion between i and j location is

$$C_{ij} = C(Y, A, d_{ij}) \quad (7)$$

where Y is amount of goods transported

A is transportation capacity(width)

d_{ij} is a measure of spatial separation between location i and j (e.g. distance)

case will happen. $\text{Min}_x (1+\mu) \{x [Qk_1 - Mt_1] + Mt_1\}$, Mt_1 is constant term

So, $Qk_1 < Mt_1$ $x^* = 1$ ie M site

$Qk_1 > Mt_1$ $x = 0$ C site

$Qk_1 = Mt_1$ any point between M and C

5) The concept of congestion can be depicted by the flow density diagram.

$\partial C/\partial Y > 0$, $\partial C/\partial A < 0$. $\partial C/\partial d < 0$ ⁶⁾.

In the literature of regulation, economic behaviors of firm are restricted by supply(output) regulation as well as rate of return regulation. Hence, there are some cases that have to produce certain level of amount of product regardless of firm's location. In other words, output is fixed or independent of location.

In case of firm output, the transport cost becomes⁷⁾

$$k(x)x + C_1 \cdot \tau \tag{8a}$$

where $C_1 \equiv C_{FC} \equiv C(Y, A, x)$, F is a firm location

C is a city, market

τ is a unit congestion price, for simplicity ($\tau=1$).

And in case of intermediate input, M the transport cost becomes

$$(y-x)t(x) + C_{MF} \cdot \tau \tag{8b}$$

When M is an intermediate input material site.

$$C_2 \equiv C_{MF} \equiv C(M, A, x) \tag{9}$$

In order to derive explicit results, let us allow some assumptions on transport costs.

Assumption 1

$$t'(x) = dt(x)/dx = t_1 > 0 \tag{10}$$

$$k'(x) = dk(x)/dx = k_1 > 0 \tag{11}$$

Assumption 2

Congestion between firm site and market, city is defined

as $C_1 = u_1 Y / A - u_2 x$

where $\partial C_1 / \partial Y = u_1 / A > 0$,

6) If the distance between i and j location increases, the road length increases. So the possibility of congestion and level of congestion decreases.

7) We assume that the effects of other firm's input/output are fixed.

$$\partial C_1 / \partial A = u_1 Y / A^2 < 0$$

$$\partial C_1 / \partial x = -u_2 < 0$$

Y : amount of good transported

A : width of roads(or number of lanes)

x : distance from origin to destination

u_1, u_2 : positive parameters

Assumption 3

Congestion between firm site and input site is defined

as $C_2 = v_1 \cdot M / v_2 x$

where $\partial C_2 / \partial M = v_1 / A > 0$,

$$\partial C_2 / \partial A = -v_1 M / A^2 < 0$$

$$\partial C_2 / \partial Cx = -v_2 < 0$$

v_1, v_2 : positive parameters

Assume production function

$$Y = f(M) = M^\alpha$$

(12)

where $\alpha > 1$

We could find the following propositions⁸⁾.

(a) If $v_2 / u_2 < M^{\alpha-1}$, then the optimum location of firm with no regulatory constraint under agglomeration and congestion moves toward a market, city comparing to the optimum location of firm with no constraint under agglomeration and no congestion

i, e

8) We analyzed the regulated firm case. We found that under scale (agglomeration) economies/diseconomies and congestion for a given K, N, M the optimum location rule of the regulated firm is the same as the one of unregulated profit maximizing firm under scale(agglomeration) economies/diseconomies and congestion. i.e. $x^* = f(Y, M)$.

However, $x^*(\cdot | R > 0, A > 0, C > 0)$ is not equal to $X^*(\cdot | R = 0, A > 0, C > 0)$

where R: regulation, A: agglomeration, C: congestion. In other words, two optimum locations are generically different each other.

$$x^*(M, t_1, k_1 \mid C > 0, A > 0)$$

$$< x^*(M, t_1, k_1 \mid C = 0, A > 0)$$

(b) $v_2/u_2 \geq M^{\alpha-1}$ then

$$x^*(\cdot \mid C > 0, A > 0) \geq x^*(\cdot \mid C = 0, A > 0)$$

We could find the proof:

$$\text{From } \partial \pi / \partial x = 0$$

$$\text{If } k(x) = k_1 \cdot x, t(x) = t_1 \cdot x$$

$$[-k_1 \cdot x - k_1 \cdot x + u_2] M^\alpha = M [-t_1 \cdot x + (1-x) \cdot t_1 + v_2]$$

Hence,

$$x^*(M, t_1, k_1 \mid C > 0, A > 0)$$

$$= \frac{Mt_1 + (M \cdot v_2 - M^\alpha u_2)}{2t_1 M - 2k_1 M^\alpha} \quad (13)$$

In case of no congestion, the optimum location becomes

$$x^*(M, t_1, k_1 \mid C = 0, A > 0)$$

$$= \frac{Mt_1}{2t_1 M - k_1 M^\alpha} \quad (14)$$

Therefore, if $v_2 = u_2$ or $(M \cdot v_2 - M^\alpha u_2) < 0$ then $x^*(M, t_1, k_1 \mid C > 0, A > 0) < x^*(M, t_1, k_1 \mid C = 0, A > 0)$

Case(b) can be proved by same ways.

Also, we could find the similar Proposition. In case of no congestion, the optimum location of the unregulated firm under agglomeration economies moves away from the market, city, as the level of agglomeration (α) is getting bigger and iff $M > 1, (2t_1 M - k_1 M^\alpha) > 0$.

For this proof we could find the followings. From $x^*(M, t_1, k_1 \mid C = 0, A > 0) = Mt_1 / (2t_1 M - k_1 M^\alpha)$, $k_1 > 0$ $k_1 M^\alpha$ is an increasing function in α ($\alpha < 1$).

Hence, as α increases, the denominator will decrease with above conditions. Now, we could check the effect of α on optimum location(x^*). Under no congestion, we have

$$x^*(\cdot | C=0, A>0) = \frac{Mt_1}{2t_1M - 2k_1M^\alpha} > 0 (\neq 0)$$

IV. Optimum Location of Firm: Expense-Preference

Model Approach

Recent trend is to diversify the research methods in order to overcome the rapidly changing socio-economic environment. Profit maximizing is not the only modern firm's objectives. The key economic issue that the organizational transformation of modern firm is associated with increasing managerial specialization and the consequent separation of the firm's management from its ownership. The divorce raises the possibility that there are some gaps between decisions of manager's decisions and the owner's profit interests [Jensen(1983), Czamanski and Fogel(1985)]. Following the evolving corporate structure of modern firm, optimal locations are a function of the extent to which ownership and management are divorced. By using the expense-preference function of firm's manager [Czmanski(1987)], we can derive the optimal location of firm under agglomeration or congestion within the above location problem framework.

A general utility function of manager can be described as

$$U = U(\Theta\pi(x), l(x)) \tag{15}$$

where $\Theta\pi$ represents the fraction Θ of firm's profits, π and $l(x)$ means consumption of leisure at x .

Assume that $\partial \pi / \partial x = \pi_x > 0$, $\partial l / \partial x = l_x < 0$

By applying same production function $Y = f(M) = M^a$ ($a > 1$) (16)

$$\pi_x = [-2k_1 \cdot x + u_2]M^a - M [t_1 - 2t_1 \cdot x + v_2]$$

$$\Theta U_1 [x \cdot (-2k_1 \cdot M^a + 2t_1 \cdot M) - M t_1 + u_2 M^a + M v_2] = -U_2 \cdot l_x$$

$$x^* = \frac{M t_1 - M v_2 - M^a u_2 - U_2 l_x / \Theta U_1}{2t_1 M - 2k_1 M^a} \quad (17)$$

Equation (17) is the optimum location of expense preference model approach.

From the profit maximizing approach, we derived the optimum location

$$x^* (\cdot | \text{P.M.}) = \frac{M t_1 - M^a u_2 + M v_2}{2t_1 M - 2k_1 M^a}$$

Therefore, we can find the difference between the two different approaches.

$$x^* (\cdot | \text{E.P.}) - x^* (\cdot | \text{P.M.}) = (-2M v_2 - U_2 l_x / \Theta U_1) / \Omega \quad (18)$$

$$\text{where } \Omega = 2t_1 M - 2k_1 M^a$$

Hence, we have

$$(i) \left(2M v_2 + \frac{U_2 l_x}{\Theta U_1} \right) / \Omega < 0$$

$$x^*(\cdot | \text{E.P.}) > x^*(\cdot | \text{P.M.})$$

Optimum location from profit maximization model is closer to market site (city) than that of expense preference model.

$$(ii) (2Mv_2 + \frac{U_2 \ell x}{\Theta U_1})/\Omega \geq 0$$

$$x^*(\cdot | E.P) \leq x^*(\cdot | P.M)$$

Optimum location from expense preference model is closer to market site than that of profit maximization model. Hence, the optimum location from profit maximization model is not always same as the optimum location of expense preference model. It depends on various parameters including α , t_1 , k_1 , etc,. From the assumption, the market means the city. Hence, there is high possibility that firms are gathering around suburban area from this result.

Because we used $k(x)$ as transport rate on the firm's product and $t(x)$ as transport rate on the firm's intermediate input, t_1 ($=t'(x)=dt(x)/dx>0$) and $k_1(=k'(x)=dk(x)/dx>0)$ are important factors deciding the optimum location.

Translating the mathematical meaning is very important in order to find policy implications. If $\Omega = 2t_1M - 2k_1M^\alpha > 0$, then the optimum location for expense preference model is closer to market site than that of profit maximizing model.

If $t_1M > k_1M^\alpha$, then this yields $(t_1/k_1)M^{1-\alpha} > 0$.

Because $(t_1/k_1) > 0$ and $M > 0$, if $\alpha > 1$, then the optimum location for expense preference model is closer to market site(city) than that of profit maximizing model.

V . Concluding Remarks

In this paper, we reviewed the optimum location of industrial city(big company town) under regulatory constraint, agglomeration and congestion. We considered the case of regulated firm under agglomeration economies/diseconomies and congestion in the economy. We used the spatial settings with linear location line. Capital is ubiquitous and labor is concentrated at

the single site. We introduced an active constraint of a fair rate of return from the pioneering work of Averch and Johnson to the location problem of the big firm with multi-inputs. With multi-inputs, simple one dimension locational problems are solved. Also, we introduced the modern firm's recent behaviour transformation related to managerial specialization and the consequent separation of the firm's management from its ownership. Hence there is a possibility of some gaps between manager's decisions and the owner's profit interests. In this paper, we focused on the locational comparison between profit maximizing approach and expense preference model approach.

The optimum location varies according to values of related parameter such as agglomeration, transport rate, etc and analytical methods such as expense preference or profit maximization approach.

Optimum location from expense preference model is closer to market site than that of profit maximization model in case of

$$(2Mv_2 + \frac{U_2 \ell_x}{\Theta U_1})/\Omega \geq 0 .$$

If $\Omega = 2t_1M - 2k_1M^\alpha > 0$, then the optimum location for expense preference model is closer to market site than that of profit maximizing model.

Because $(t_1/k_1) > 0$ and $M > 0$, if $\alpha > 1$, then the optimum location for expense preference model is closer to market site(city) than that of profit maximizing model. It is believed that the optimum firm location under various environments including congestion, agglomeration, regulations is varying according to the various parameters such as α , t_1 , k_1 , etc,. Because we assumed that the market means the city, with the introduction of expense preference approach the phenomena that modern firms are gathering around suburban area could be explained.

Because we used restricted assumptions on spatial settings, there are various possibility to exclude the real policy problems such as complicated tax, subsidies and so on. For future extension of this research, it seems to me that firm location

impact on land price, and introduction of uncertainty of output price, input price under agglomeration, congestion will be possible. Also, empirical test can be implemented with real data within this framework.

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<Appendix>

Process of finding firm's optimum location of profit maximizing
(From eq. 5)

Assuming that the rate of return constraint is binding, the Lagrangian becomes

$$H = [P(Y) - k(x)x - C_1]A(N)f(N, K, M) - M[(y-x)t(x) + C_2 + P] - wN - rK - \mu \{ [P(Y) - k(x)x - C_1]A(N)f(N, K, M) - M[(y-x)t(x) + C_2 + P]wN - gK \} \quad (19)$$

$$\partial H / \partial K = (1 - \mu)[G - K(x)x - C_{1K}]Y_K - r + \mu g = 0 \quad (19a)$$

$$\partial H / \partial N = (1 - \mu)[G - k(x)x - C_{1N}]Y_N - (1 - \mu)w = 0 \quad (19b)$$

$$\partial H / \partial M = (1 - \mu)[G - k(x)x - C_{1M}]Y_M - (1 - \mu)[(y-x)t(x) + C_2 + P + C_2M] = 0 \quad (19c)$$

$$\partial H / \partial x = -(1 - \mu)[k(x) + k'(x)x + C_{1x}]Y - (1 - \mu)[-t(x) + (y-x)t'(x) + C_2x]M = 0 \quad (19d)$$

$$\partial H / \partial \mu = -\{ [P(Y) - k(x)x - C_1]A(N)f(N, K, M) - M[(y-x)t(x) + C_2 + P] - wN - gK \} = 0$$

where $G = P(Y) + P'(Y)Y$

$$t'(x) = t(x)/x$$

$$k'(x) = k(x)/x$$

Also, this maximizing problem requires $D > 0$, $D_1 < 0$. Assume that there exists finite optimal values N^* , K^* , M^* and x^* for this nonconstrained

maximization problem.

Then, the nonconstrained rate of return on capital for the nonconstrained case can be computed by

$$r^* = \{ [P - k(x^*)x^* - C_1] A(n^*) f(n^*, k^*, m^*) - M^* [(y - x^*)t(x^*) + C_2 + P] - wN^* \} / k^* \quad (20)$$

For the proof we could find the followings. Given, K, N, and M from $\partial H / \partial x$ equation,

$$\text{we have } [k(x) + k'(x)x + C_1x]Y = -[-t(x) + (y-x)t'(x) + C_2x]M$$

above equation is exactly the same as $\partial H / \partial x$ equation under $\mu=0$ where is no regulatory constraint. Therefore, the optimal location rule is unaffected whether the firm is regulated or not and whether scale economies/diseconomies (agglomeration) and congestion exists or not. Y and M are different both in the regulated case and unregulated case.

Assuming that the rate-of-return constraint is binding so that it is treated as an equality, the Lagrangian function is

$$H = [P(Y) - k(x)x] A(N) N^a K^b M^{1-a-b} - M [(y-x)t(x) + P] - wN - rK - \mu \{ [P(Y) - k(x)x] A(N) N^a K^b M^{1-a-b} - M [(y-x)t(x) + P] - wN - gK \} \quad (21)$$

$$\partial H / \partial K = (1-\mu) [G - k(x)x] Y_K - r + \mu g = 0 \quad (21a)$$

$$\partial H / \partial N = (1-\mu) [G - k(x)x] Y_N - w + \mu w = 0 \quad (21b)$$

$$\partial H / \partial M = (1-\mu) [G - k(x)x] Y_M - (1-\mu) [(y-x)t(x) + P] = 0 \quad (21c)$$

$$\partial H / \partial x = (1-\mu) [k(x)x + k'(x)x] Y - (1-\mu) [-t(x) + (y-x)t'(x)] M = 0 \quad (21d)$$

$$\partial H / \partial \mu = \{ [P(Y) - k(x)x] Y - M [(y-x)t(x) + P] - wN - gK \} = 0 \quad (21e)$$

where $G = P(Y) + P'(Y)Y$

$$t'(x) = \partial t(x) / \partial x$$

$$k'(x) = \partial k(x) / \partial x$$

If there is no rate of return, or if the constraint is not active(i. e. $\mu=0$), Equations 19(a)(b)(c)(d) become

$$[G-k(x)x] Y_K - r = 0 \quad (22a)$$

$$[G-k(x)x] Y_n - w = 0 \quad (22b)$$

$$[G-k(x)x] Y_M - [(y-x)t(x)+P] = 0 \quad (22c)$$

$$- [k'(x)+k''(x)x] Y - [-t(x)+(y-x)t'(x)] M = 0 \quad (22d)$$

Assume that there exists finite optimal values N^* , K^* , M^* AND x^* for this nonconstrained maximization problem.

Then, the nonconstrained rate of return on capital for the nonconstrained case can be computed by

$$r^* = \{ [P-k(x^*)x^*] A(N^*)f(N^*,K^*,M^*) - M^* [(y-x^*)t(x^*)+P] - wN^* \} / K^* \quad (23)$$

초록

기업의 최적 입지 분석
-수익 극대화 및 지출선호 모형을 중심으로-

양지청 (국토연구원)

이 연구는 기업 또는 산업의 최적입지 비교 분석에 관한 연구이다. 최근에는 어떤 지역에서 생산하느냐에 따라 기업(산업)의 생존에도 영향을 미치는 것으로 분석되고 있다. 잘 알려진 바대로 집적경제, 혼잡효과, 산업의 규제 등이 있을 때 최적입지의 결정은 다소 복잡한 메카니즘에 의해 결정되지만 단순 선형입지 공간구조아래서 상호 비교분석 할 수 있으며 구체적 산업, 도입된 가정을 완화 해 가면서 정책분석의 도구로 활용할 수 있다. 양지청(1993)의 연구는 규제받는 기업의 최적 입지는 규제받지 않는 기업과 집적경제, 혼잡효과가 존재할 시에 동일한 규칙(rule)에 의해 결정되어지나 속성적으로 다르다(generically different)는 것을 밝힌바 있다. 이 연구에서는 먼저 지출선호 모형에 의한 기업의 최적입지를 도출하였다. 그리고 최적입지는 이윤극대화 모형으로 접근할 경우와 지출선호모형으로 접근할 경우 서로 상이 할 수 있음을 보여주고 있다. 즉, 이 논문의 주요 내용은 집적경제, 혼잡효과 등이 존재할 시 기존 방법에 따라 이윤극대화 모형에 의한 기업의 최적입지를 도출하고, 지출선호모형에 의한 최적입지를 도출하여 그 차이점을 분석하는 것이다. 이와 같이 여러 가지 조건 하에서 최적입지는 관련 변수에 의해 결정됨을 알 수 있다. 주로 Market 쪽으로 기업이 입지할 가능성이 큼을 제시해 주고 있다. 이는 도시 주변지역으로 집중될 가능성을 설명해주는 것이라 할 수 있다. 복잡한 현실세계를 단순화 해 진행하는 이론적 접근으로 정책적 제안측면에서 한계가 존재하기도 하지만, 규제 받지 않는 기업(산업)이외에, 다양한 규제를 받고 있는 기업(산업)도 많이 있기 때문에 규제(R), 집적경제(A), 혼잡효과(C) 등을 조정해서 상호 영향을 분석할 수 있을 것이다.

* Keyword: 최적입지, 기업, 산업 최적입지 비교분석, 집적경제, 규제된 산업
지출선호 모형