The Test of Regional Difference of Public Capital Effect on the Private Sector Output in the U.S.

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I. Introduction

Economic theory has long been fascinated with the production of goods which are characterized as public in nature. While examples of pure public goods are difficult to identify, in practice and in the literature, the set of goods labeled as public typically includes highways and transportation infrastructure, education, sewers, sewage systems and utilities. It is in the production of and spending on these goods that local, state and federal governments largely justify the taxation of their citizenry. This spending has come to be known as public capital investment. The definition of public capital used in this paper is the same definition that used in Holtz-Eakin(1993).

A central issue in the study of public capital, the issue addressed in this paper, revolves around the question of whether public capital contributes to the production of private output. Despite clear theoretical links between public capital investment and productivity growth, empirical evidence is mixed. Aschauer (1989a, 1989b) and Munnell (1990a, 1990b) find evidence to suggest that public capital investment raises economic output. By contrast, Holtz-Eakin (1993), Hulten and Schwab (1984), and Eberts (1990a and 1990b) find no significant effect of public capital on output.

This study unveils the reason why their results are different and the output elasticity of each type of public capital in each industry in the Sun and Snow belts. To elucidate the effect of public capital on output, previous works utilized the econometric method or Solow's growth accounting technique. The studies incorporating Solow's growth accounting technique measured multifactor productivity growth rates assuming perfectly competitive input markets and constant returns to scale. By testing the correlation between the estimated multifactor productivity growth rate and public capital growth rate, Hulten and Schwab (1984) showed the weak link between multifactor productivity growth and public capital. Eberts (1990b) also argued that the effect of public capital on productivity was trivial.

However, there are two main drawbacks to the Solow's technique. First, the model assumes that all input markets are perfectly competitive. If any input market is less than perfectly competitive, income shares do not equal factor shares and estimates of multifactor productivity may be biased.Thus, there have been some studies to control this assumption such as Hulten and Schwab (1991). Second, since the multifactor productivity growth rate is measured as a residual, this estimate is affected by all other factors including public capital, only excluding private capital and labor. Since they couldn't separate the change in multifactor productivity that was affected only by public capital, the simple correlation between multifactor productivity and public capital may be biased because multifactor productivity estimated may be affected by the other multifactors. To test for the true effect of public capital on multifactor productivity, it is necessary to estimate the multifactor productivity growth rate that is affected only by public capital.

To solve the first problem, this study utilizes the Malmquist Index for measuring multifactor productivity. This method does not require the perfectly competitive input markets assumption. To solve the omitted variable bias problem, this study estimated two types of multifactor productivity growth rates. The first type was a multifactor productivity growth rate affected by all the multifactor entities including public capital and excluding only private capital and labor. The second involved a multifactor productivity growth rate affected by all the multifactors except one of each type of public capital as well as private capital and labor. The difference resulting from these two multifactor productivity growth rates is the change in multifactor productivity growth affected only by one type of public capital.

The study also examines whether public capital productivity varies by the type of public capital. It disaggregates public capital investment into four separate categories; education, highways and streets, sewerage and utilities. We look for differences in the productivity of public capital in economic output as a whole and in four broadly defined industries; agriculture, manufacturing, non-farming and non-manufacturing. Finally, the study tests whether the effect of public capital varies in the southern and northern U.S., the Sun and Snow Belts.

The study finds that,on average, investment in public capital statistically and significantly increases output growth rates. These effects vary substantially across industries and by the type of public capital investment. However, differences in output growth rates of the Sun and Snow Belts cannot be attributed to differences in the productivity of public capital.

The rest of the paper proceeds as follows: In section II, the model is presented. The data and results are discussed in sections III and IV respectively. A brief conclusion closes the paper.

II. Method

1. Malmquist Index

Multifactor productivity growth rates are analyzed using the Malmquist Index. The analysis takes place in three steps. First, observed input-output bundles are sorted in terms of input-output combinations. Under the assumption that production exhibits non-increasing returns to scale, a "grand production possibility frontier"is non-parametricly defined by the set of input-output bundles with the greatest observed output for a given level of inputs. As noted above, this method does not require input markets to be perfectly competitive so all observed input-output bundles must not lie on the production possibility frontier. Once the grand production possibility frontier (PPF) is determined, the method of Fare et. al (1994) is used to calculate distance functions that measure differences between observed input-output bundles and those judged to be on the frontier of production efficiency, i.e., the difference between each input output combination and the grand PPF for the associated input level. Using linear programming, the measurement of the distance estimates assumed non-increasing returns to the scale of grand production possibility frontier. The distance function in state j is given by:

$$D(y^j, x^j) = \max$$

$$\text{s.t. } y_j \quad z_j y_j$$

$$z_j x_{jm} \quad x_{jm}\; m=1,2,......M$$

$$z_j \quad 1$$

$$z_j \quad 0, j=1,2,......J$$

where y denotes output, x the vector of inputs, m the number of inputs (m=1,2,......M), j denotes state activities (j=1,2,......J), z the vector of intensity variables (z_j 0) which allows for the expansion or contraction of observed activities radially for the purpose of forming a grand production possibility frontier.(Färe et. al (1994)) The estimated distances are then put into the algorithm developed by Färe et. al (1994)However, they regard all production bundles lower than the production possibility frontier as inefficient productions even though it may result from the smalleramount of multifactors involved (including public capital) in comparing with other states. to derive a value of the Malmquist Index for each input-output combination. The Malmquist Index is given by:

$$M = \left[\left(\frac{D_t(Y_t, X_t)}{D_t(Y_{t+1}, X_{t+1})} \right) \times \left(\frac{D_{t+1}(Y_t, X_t)}{D_{t+1}(Y_{t+1}, X_{t+1})} \right) \right]^{\frac{1}{2}} \qquad (1)$$

where D_t (.) denotes the distance function from the grand production possibility frontier at time *t*, D_{t+1}(.) denotes the distance function from the grand production possibility frontier at time *t+1*. (Y_t, X_t) denotes the observed production combination at time *t*, and (Y_{t+1}, X_{t+1}) denotes the observed production combination at time *t+1*. Thus, $D_t(Y_t, X_t)$ is the distance function estimate between the observed output Y_t and the output on the grand production possibility frontier at time *t* for the given input X_t. $D_{t+1}(Y_t, X_t)$ is the distance function estimate between the observed output $Y_t$and the output on the grand production possibility frontier at time *t+1* for the given input X_t. The other distance functions are similarly defined.

Equation (1) can be transformed to the following Detailed explanations of the process can be found in R. Färe, S. Grosskopf, M. Norris and Z. Zhang(1994). If we use the perfect input market assumption from Solow's technique in the Malmquist Index, the distance estimates will always equal 1 (i.e., $D_t(y_t, x_t)=1$ and $D_{t+1}(y_{t+1}, x_{t+1})=1$) and then the equation becomes [A(t+1)/A(t)] which is the same equation as the growth of multifactor productivity equation from Solow's growth accounting technique. However, if any state produces output inefficiently, or if the amount of multifactor used is different across states, then the observed activity will be located below the grand production possibility frontier. In this case, the distance functions $D_t(y_t, x_t)$ and $D_{t+1}(y_{t+1}, x_{t+1})$ are not equal to one and Solow's growth accounting index would be a biased estimate of multifactor productivity change.

$$M = \left[\left(\frac{A(t+1)}{A(t)} \right) \times \left(\frac{D_t(Y_t, X_t)}{D_{t+1}(Y_{t+1}, X_{t+1})} \right) \right] \qquad (2)$$

This is the multifactor productivity growth rate at time t.

2. The Model for Measuring Public Capital Effects

The multifactor productivity growth rates estimated with the above method can be used to investigate whether productivity growth varies across public capital of different types. The simple method used here examines how estimated multifactor productivity growth rates change as investment of specific types of public capital issuccessively introduced into the model. In practice, we estimate multifactor productivity growth rates using series of production functions that successively include specific types of public capital as distinct inputs into production. The first production function sweeps all public capital into the multifactor (denoted $A(MFP)$). Therefore, the common formulation of the production function is

$$Y_t = A(MFP_t)f(L_t, K_t^{pri}) \tag{3}$$

where Y_t = output, L_t = labor input, K_t^{pri} = private capital input, and

$A(MFP_t)$ =multifactor productivity at time t .

Taking the differential with respect to time and dividing by Y_t yields:

$$\frac{\dot{Y}_t}{Y_t} = \frac{A(M\dot{F}P_t)}{A(MFP_t)} + \frac{\Delta f(L_t, K_t^{pri})}{\Delta L} * \frac{L_t}{f(L_t, K_t^{pri})} * \frac{\dot{L}_t}{L_t} + \frac{\Delta f(L_t, K_t^{pri})}{\Delta K^{pri}} * \frac{K_t^{pri}}{f(L_t, K_t^{pri})} \frac{\dot{K}_t^{pri}}{K_t^{pri}}$$

where $\frac{\dot{Y}_t}{Y_t}$ = % growth in Y , $\frac{M\dot{F}P_t^1}{MFP_t^1}$ = % growth in MFP_t^1 ,

$\frac{\dot{L}_t}{L_t}$ = % growth in L , $\frac{\dot{K}_t^{pri}}{K_t^{pri}}$ = % growth in K_t^{pri} at time t .

This equation can thus be expressed for each year as,

$$\frac{\dot{Y}_t}{Y_t} = \frac{A(M\dot{F}P_t)}{A(MFP_t)} + S_l * \frac{\dot{L}_t}{L_t} + S_{pri} * \frac{\dot{K}_t^{pri}}{K_t^{pri}}$$

where S_l = the output elasticity of labor input

S_{pri} = the output elasticity of private capital input.

Rearranging terms yields:

$$\frac{\overset{\bullet}{A(MFP_t)}}{A(MFP_t)} = \frac{\dot{Y}_t}{Y_t} - S_l * \frac{\dot{L}_t}{L_t} - S_{pri} * \frac{\dot{K}_t^{pri}}{K_t^{pri}} \qquad (4)$$

In order to set the production functions that exclude the effect of only one type of public capital $p(K^p)$ from the other factors(MFP^p) multifactor productivity(MFP) , I redefine the multifactor ($A(MFP)$) as follows:

$$A(MFP) = B(MFP^p)C(K^p)$$

where K^p: one type of public capital (education, highways and streets, sewerage, utilities)

Then, the common formulation of the production function(equation(3)) can be rearranged as follows:

$$Y_t = B(MFP^p)C(K^p)f(L_t, K_t^{pri}) \qquad (5)$$

Letting $g(L_t, K_t^{pri}, K_t^p) \equiv C(K^p)f(L_t, K_t^{pri})$, the production function is

$$Y_t = B(MFP^p)g(L_t, K_t^{pri}, K_t^p) \qquad (6)$$

The superscripts *pri* and *p* represent private capital and one type of public capital respectively and $B(MFP^p)$ stands for the measured multifactor productivity that excludes the effect of public capital of type p. The category of public capital includes investment in education, highways and streets, sewerage, utilities, or it represents the aggregate investment for all types.

Taking the differential with respect to time and dividing by Y_t yields:

$$\frac{\dot{Y}_t}{Y_t} = \frac{B(\dot{MFP}_t^p)}{B(MFP_t^p)} + \frac{\Delta f(L_t, K_t^{pri}, K_t^p)}{\Delta L} * \frac{L_t}{f(L_t, K_t^{pri}, K_t^p)} * \frac{\dot{L}_t}{L_t} + \frac{\Delta f(L_t, K_t^{pri}, K_t^p)}{\Delta K^{pri}} * \frac{K_t^{pri}}{f(L_t, K_t^{pri}, K_t^p)} \frac{\dot{K}_t^{pri}}{K_t^{pri}}$$
$$+ \frac{\Delta f(L_t, K_t^{pri}, K_t^p)}{\Delta K^p} * \frac{K_t^p}{f(L_t, K_t^{pri}, K_t^p)} \frac{\dot{K}_t^p}{K_t^p}$$

where $\frac{\dot{K}_t^p}{K_t^p}$ = % growth in public capital type p at time t.

This equation can thus be expressed for each year as:

$$\frac{\dot{Y}_t}{Y_t} = \frac{B(\dot{MFP}_t^p)}{B(MFP_t^p)} + S_l * \frac{\dot{L}_t}{L_t} + S_{pri} * \frac{\dot{K}_t^{pri}}{K_t^{pri}} + S_p * \frac{\dot{K}_t^p}{K_t^p}$$

where S_p = the output elasticity of one type(p) of public capital input.

Rearranging terms yields:

$$\frac{B(\dot{MFP}_t^p)}{B(MFP_t^p)} = \frac{\dot{Y}_t}{Y_t} - S_l * \frac{\dot{L}_t}{L_t} - S_{pri} * \frac{\dot{K}_t^{pri}}{K_t^{pri}} - S_p * \frac{\dot{K}_t^p}{K_t^p} \qquad (7)$$

By equating the two growth rates equations (4) and (7) yield the following equation :

$$\frac{A(\dot{MFP}_t)}{A(MFP_t)} - \frac{B(\dot{MFP}_t^p)}{B(MFP_t^p)} = S_p * \frac{\dot{K}_t^p}{K_t^p} \qquad (8)$$

Using the Malmquist Index, the multifactor productivity growth rates($\frac{A(\dot{MFP}_t)}{A(MFP_t)}$ and four types of $\frac{B(\dot{MFP}_t^p)}{B(MFP_t^p)}$) can be estimated. Data on changes in public capital investment over time for each state and in different industries can be used to derive the ratio on the right hand side of equation (8).

3. Empirical Model

In order to compare the effects of public capital on productivity in the Sun Belt and Snow Belt, the statistical version of equation (8) is specified as:

$$\frac{A(\dot{MFP}_t^p)}{A(MFP_t^p)} - \frac{B(\dot{MFP}_t^p)}{B(MFP_t^p)} = \alpha + dum1 + \beta_{p1}\frac{\dot{K}_t^p}{K_t^p} + \beta_{p2} * dum1 * \frac{\dot{K}_t^p}{K_t^p} + D_s + D_t + \varepsilon \qquad (9)$$

where the rates of change over time are defined as above. The variable *dum1*is an indicator variable that equals 1 when the observed state belongs to the Snow belt, otherwise, it is zero, *D* is a vector of state and year dummy variables.Holtz-Eakin (1993) show that estimates of output elasticity of each type of public capital varies greatly when state and year effects are not held constant. is an error term which is assumed to be *iid*(independently and identically distributed).

$_{p1}$ is the output elasticity of public capital *p* in the Sun belt and $_{p2}$ is the difference of estimated output elasticity between the Sun and Snow belts. The sum of $_{p1}$ and $_{p2}$ is the estimated output elasticity in the Snow belt.

III. Data

The data consist of annual observations of industrial output, total employment, private and public capital from the 48 contiguous states from 1969 to 1986. All dollar figures are reported in 1982 constant dollars. Data were analyzed separately for three broad industrial sectors: agriculture, manufacturing, and non-farm, non-manufacturing.The non-farm, non-manufacturing sector includes consists of mining, construction, transportation, communication, public utilities, wholesale trade, retail trade, finance, insurance, real estate, and other related services.

Data from the Bureau of Economic Analysis (BEA) were used to measure output and employment. Gross state product (GSP) was used as the measure of output for each industry. Employment was measured by average annual employment in each industry. State-level data on the stock of private capital and public capital are not available. Munnell estimates the stock of private capital in each state and year by apportioning BEA national stock estimates of various sectors among the states. The majority of the data, which was used as proxies for each industry were derived from economic censuses. Because the census is published every five years, the data from the particular census year were used to calculate BEA assets for the year of the study, as well as for preceding years and following years as the state's share of the proxies. Detailed explanation of the method used to approximate private capital stocks can be found in Munnell[12]. Douglas Holtz-Eakin (1993) estimates public capital stocks using estimates of aggregate capital accumulations and capital for each specific governmental function including education, highways and streets, sewerage, and utilities. The data utilized to calculate the various types of public capital was obtained from Governmental Finances and State Governmental Finances which derive data information on annual investment flows for each state. Using the annual investment flows data, each capital stock by incorporating the perpetual inventory technique. Detailed explanation of the method used to approximate public capital stocks can be found in Holtz-Eakin[7].

IV. Results

1. Total industry

Table 1 indicates the output elasticity of each type of public capital for the industry as a whole. This result supports Munnell's results even though the estimated coefficient is a little less. Munnell showed that the output elasticity of total public capital ranges from 0.37 to 0.49. Since Munnell did not control for state-specific characteristics, as Douglas Holtz-Eakin (1994) pointed out, the magnitude of the estimated coefficient may be somewhat exaggerated by those characteristics. But contrary to Douglas Holtz-Eakin's results (1994), the magnitude decrease of output elasticity was relatively minor and it did not close at zero.

Table 1. The output elasticity in total industry

	Highways	**Sewerage**	**Education**	**Utilities**
Sun belt	**0.11(3.50)**	**0.36(24.48)**	**0.241(7.76)**	**0.03(4.46)**
Snow belt	**0.05(3.96)**	**0.18(12.36)**	**0.240(5.64)**	**0.10(4.94)**
p2	**-0.06(-1.89)**	**-0.18(-8.20)**	**-0.001(-0.02)**	**0.07(3.56)**

* The figures in parentheses are t-statistics

The table also shows the estimated elasticity and the magnitude difference of the estimated elasticities between the Sun and Snow belts. For highways and streets, and sewerage capitals, the estimated output elasticities in the Sun belt were larger than those in the Snow belt, and the magnitude difference in the estimated output elasticities($_{p2}$) were quite significant. The reasoning behind the resulting small magnitudes in regards to highway and street effects compared with those of other types of capital is that highways were already built previous to the duration of this study and so, the effect of additional new investments proved to be rather small. For educational capital, the estimated output elasticities in both regions were large, and these magnitudes were not significantly different. For utilities capital, the estimated elasticities in both regions were small. However, the magnitude differences of the estimated elasticities were significant.

2. Agriculture

In agriculture, the direct effect of total public capital decreased greatly compared with that of the total industry and the estimated output elasticity was statistically insignificant. The effects of Highways and streets, Sewerage, and Utilities capitals were a bit greater while the effect of Educational capitals was near zero. The magnitude difference of estimated elasticities of all types of public capitals between the Sun and Snow belts in agriculture were insignificant.

The change in each type of public capital will affect the output in all industries and this output change in one industry will affect the output in other industries again. In the total industry analysis, this indirect effect also affects the output elasticity measurement. However, in the disaggregate analysis, when the output elasticity in each industry is measured, the effect of other industries'outputs resulting from the change of public capital on the output elasticity is excluded. That is, the output elasticity, in each industry analysis, measures only the direct effect of each type of public capital and so, the output elasticity of each industry may be less than that in the total industry figures.

Table 2. The output elasticity in agriculture

	Highways	**Sewerage**	**Education**	**Utilities**
Sun belt	**0.05(0.99)**	**0.09 (4.71)**	**0.03(0.87)**	**0.05(1.55)**
Snow belt	**0.11(2.67)**	**0.11(4.40)**	**0.06(0.84)**	**0.13(3.00)**
$_{p2}$	**0.06(0.87)**	**0.02(0.45)**	**0.03(0.48)**	**0.08(1.31)**

3. Manufacturing

In Manufacturing, the results display a positive effect of public capital on output. This is different from the results of Hulten and Schwab (1984) and Eberts (1990b)that showed an insignificant effect of public capital on output. Thus, their results may be biased because they tested the effect using Solow's technique assuming perfectly competitive input markets and using the average productivity growth affected by all multifactors.

The effect of Sewerage was the greatest among all public capitals and the rest of the public capitals had smaller estimated elasticities.

In the analysis of regional differences between the Sun and Snow belts, the magnitude differences of estimated elasticities for all types of public capitals were insignificant except that of utilities. The estimated elasticity of utilities capital in the Snow belt was significantly and slightly larger than that of the Sun belt.

Table 3. The output elasticity in manufacturing

	Highways	**Sewerage**	**Education**	**Utilities**
Sun belt	**0.07(1.95)**	**0.23(11.17)**	**0.06(2.77)**	**0.02(2.11)**
Snow belt	**0.08(1.76)**	**0.24(10.70)**	**0.14(2.20)**	**0.09(2.49)**
p2	**0.01(0.15)**	**0.01(0.61)**	**0.08(1.43)**	**0.07(2.14)**

4. Non-farm and non-manufacturing

In non-farm, non-manufacturing, the effects of sewerage and education capitals were great while the effects of highways and streets and utilities capitals were smaller. The estimated output elasticity of sewerage in the Sun belt was significantly larger than that of the Snow belt. For utilities capital, the estimated output elasticity in the Snow belt was significantly larger than that of the Sun belt. In the case of highways and streets, and education capitals, the magnitude differences were insignificant.

Table 4. The output elasticity in non-farm and non-manufacturing

	Highways	**Sewerage**	**Education**	**Utilities**
Sun belt	**0.09(2.79)**	**0.36(19.67)**	**0.17(7.46)**	**0.04(4.40)**
Snow belt	**0.04(3.44)**	**0.13(11.49)**	**0.18(5.05)**	**0.09(4.80)**
p2	**-0.05(-1.65)**	**-0.23(-9.42)**	**0.01(0.36)**	**0.05(2.38)**

V. Conclusion

In this paper, I estimated the output elasticities of each public capital in each industry using the estimated two types of disaggregated multifactor productivity growth rates. The estimated results suggestsome important points. First of all, the output elasticity of public capital is positive and significant, and thus, results from previous research which indicated small and insignificant effects of total public capital on output using Solow's technique may be the result of using biased multifactor productivity growth rates.

Secondly, the results show that the estimated output elasticity of decomposed public capital and industry is absolutely different from the estimated results using aggregate public capital and total industry. Thus, to estimate the exact contribution of each public capital on output, the disaggregate data must be used as much as possible.

In the test of the magnitude differences in estimated elasticities of each type of public capital between the Sun and Snow belts, the results don't always support the argument that the effect of public capital on the Sun belt is much larger than that of the Snow belt. In some cases, the estimated output elasticities in the Sun belt were significantly larger than those in the Snow belt, and in other cases, those in the Snow belt were significantly larger than those in the Sun belt, or there were no significant differences between them. From these results, it can be concluded that the contribution amount of each public capital on output in each industry in both regions was different, and that the productivity growth of public capital in the Sun belt was not always larger than that of the Snow belt.

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초록

공공자본의 산출량 증대효과의 지역별 차이 분석

민동기

주요단어: 생산성, 공공자본, *Malmquist index*

본 연구에서는 미국을 대상으로 공공자본이 지역별 산출량에 미치는 효과를 비교 분석한다. 이 분석을 위한 공공자본의 생산성 자료 구축은 Malmquist Index를 사용하여 추정하였다. 기존의 보편적인 생산성 추정 방법으로는 모든 시장의 완전경쟁을 가정한 Solow 의 성장회계모형에 근거하였다. 이 모형에서는 모든 지역의 생산 활동이 효율적으로 이루어진다고 가정하지만 일부 투입요소의 외부효과와 비이동성은 이러한 가정 하에 측정된 총요소 생산성 증가율이 왜곡될 수 있음을 보인다. Malmquist Index 기법은 이러한 지역별 산출이 효율적으로 이루어진다는 가정을 전제하지 않으며 지역별 투입량 및 산출량 자료만을 이용하여 총요소생산성 증가율을 추정한다. 이 기법을 이용하여 공공자본의 생산성 증가율을 추정하는 방법은 개별 공공자본(도로, 상 하수도, 교육, 전기 전력)이 총요소에 포함된 경우와 포함되지 않은 경우의 총요소생

산 증가율의 차이를 이용하여 도출한다. 공공자본의 산출량 증대효과에 대한 논쟁이 많았지만 본 연구 결과에 의하면 대부분의 개별 공공자본들이 전체 산업뿐 아니라 각 산업의 산출량 증대에 효과가 있음을 보여주고 있어 기존 연구에서 공공자본의 산출량 증대효과가 없다는 결과는 공공자본의 생산성 증가율 추정상의 문제점에 기인한다고 볼 수 있다.

전 산업의 산출량에 대한 개별 공공자본의 탄력성 추정치를 보면 상 하수도 부문의 추정치가 가장 크게 나타나며 도로의 경우에는 상대적으로 탄력성 추정치가 작다. 이는 도로의 산출량에 대한 효과가 작다기 보다는 이미 도로가 완비된 미국의 경우 추가적인 도로에 대한 한계투자가 산출량 증대에 미치는 효과가 상 하수도 부문에 비하여 작다는 것이다.

산업별 산출량에 대한 개별 공공자본의 탄력성 추정치를 보면 전반적으로 서비스 업종과 제조업종에 대한 효과가 크게 나타났으며 농업부문에 대한 효과는 상대적으로 작게 나타났다.

지역별로 공공자본의 산출량 증대 효과의 차이를 분석한 연구결과에서는 큰 차이를 보이지 않는다.