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Appraisal of Revenue Cap Agreement in BOT Project Finance

BOT 프로젝트 파이낸스에서 운영수익상한조항의 가치평가에 관한 연구

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Contents

I. Introduction

Limited public funds to provide infrastructures have had the governments consider joining the private developer in a project finance scheme(Jun and Sharkawy. 2008). The project finance scheme followed by the stakeholders is regarded effective since it is based on the notion that the specific stakeholder, in implementing a project, who can best treat with the specific risk should be in charge of it. Because the risk evolution that usually takes place in a project finance is complex so that the project stakeholders predict, the government and the developer become closer to hedge risks they may encounter through the comprehensive cooperation process(Augenblick and Custer. 1990). The BOT scheme, which is one of the most frequently used type in unsolicited project finances, works with the way that the ownership is transferred from the developer to the government upon the concession.

The BOT project finance entails lots of unique characteristics and these characteristics can fall into two folds. The one is uncertainty and the other is managerial flexibility(Dixit and Pindyck. 1994; Moon. 2002; Trigeorgis. 1999). First, the “uncertainty” in finance, generally, represents the uncertain change of the cash flow that the project will generate. Therefore, the use of uncertainty in real asset can be thought as the uncertain cash flows unidentical to what already expected from the financial analysis. Second, the “managerial flexibility” is kind of event or reaction predetermined in a form of managerial decision change against unpredictable events during the project, specific contracts agreed on, or financial structure. Second, the “managerial flexibility” is kind of event or reaction predetermined in a form of managerial decision change against unpredictable events or financial structure during the project. The combined effect of both of “uncertainty” and “managerial flexibility” induces the change of project's cash flows described as an asymmetric payoff(Jun and Kim, 2009). The BOT project finance's unique characteristics such as huge project size, long concession period, or contractual complexity could be sources to create this asymmetric payoff and the fact that one of the most popular capital budgeting theories, NPV(Net Present Value) analysis, is limited to assess it makes people seek alternations to resolve this problem(Jun and Kim. 2009; Walker and Smith. 1995).

Amongst various managerial flexibilities in BOT projects, “Revenue Cap(RCP in this paper)” agreement used to protect the government from the developer's revenue exploitation, is considered as the most representative “risk-reward” agreement by being simultaneously applied with “MRG(Minimum Revenue Guarantee)” agreement in a BOT project(Cheah and Liu. 2006). Although RCP is very often applied in BOT projects for the government to avoid revenue exploitation, to have capital sources in financing toll system related facilities, or to reduce user's fees, the quantitative impact of the RCP agreement on the government, the developer, and the toll rate from the financial point of view are not well identified(Cheah and Liu. 2006; Jun. 2008; Mason and Baldwin. 1988). In addition, although the BOT developer seems delicate to the level of the RCP agreement because the developer's revenue can vary over the RCP level, lack of efforts to assess it may keep the BOT developer be placed under a handicap in a bidding process. Like in use of other traditional capital budgeting theories in BOT projects, NPV has also its limitation to value the RCP and the research concerning the evaluation of real assets such as real estate or infrastructure development under the managerial flexibilities have been that few(Amram and Kulatilaka. 1999; Insley and Wirjanto. 2006).

However, although there exist some problems in use of NPV analysis, it is fortune to have the alternation, thanks to the improved modern financial theory so called “option pricing theory”, to assess the complexly combined managerial flexibilities(Copeland and Antikarov. 2001). Similarities in evaluating process between the real assets and financial derivatives based on the option pricing theory can help assess the asymmetric payoff stemming from the contingency of RCP agreement in a BOT project(Mello and Pyo. 2003; Miller and Park. 2002).

So, the purpose of this paper is to quantitatively assess the impact of the RCP agreement on the project value through the numerically-developed option pricing framework on the basis of the binomial model(discrete-time approach), thereby being able to show a practical and theoretical process to quantitatively evaluate the RCP agreement in a BOT project. To justify the applicability of the approach, the hypothetical BOT toll road is used and the results obtained from the model are scrutinized over those by the traditional capital budgeting theories.

II. Theories

1. Traditional Capital Budgeting Theory: NPV Analysis

NPV analysis works well while the cash flows and the risks involved in an asset remain stable as the project goes on(Dixit and Pindyck. 1994; Jun and Kim. 2009; Luehrman. 1997; Myers. 1984). But, projects often create contingencies such as delaying, abandoning or expanding the projects by the managerial decision changes that affect future cash flows where NPV analysis may either ignore or underestimate the possibilities of the increasing project value(Trigeorgis. 1999; Myers. 1984). The way the NPV analysis recognizes risk is to reflect it through a risk-adjusted discount rate to calculate the net present value of the project(Jun. 2008; Meredith and Mantel. 1995). Each firm has its own guideline to assign appropriate risk-adjusted discount rate corresponding to their risk prereference and risks involved in different projects based on management's experience taken from various know-how and information(Trigeorgis. 1999).

Despite the fact that the NPV analysis is widely considered effective to assess project's financial feasibility, there still exists some controversies needed to be better explained. The first is this method assumes the cash outflows in different time periods but in time "0" have the same risk characteristic as the cash inflows(Jun and Sharkawy. 2008). This assumption may cause the underestimation of the risks involved because the uncertainty can take place even in cash outflows. Second, while applied in infrastructures, it can not properly assess managerial flexibility to be fit with varying decision-making when, as uncertainty is resolved, future events differ from what expected at the financial analysis(Copeland and Antikarov. 2001; Dixit and Pindyck. 1994; Luehrman. 1997; Myers. 1984; Trigeorgis. 1999). When a project is under high uncertainty, if the project entails the managerial flexibilities, it may deserve to invest even though NPV is negative. Hence, mistakenly ignoring the operating and managerial flexibilities in a project can significantly underestimate its value(Mason and Merton. 1985).

For these reasons, the real options analysis based on the financial option pricing theory is suggested by some researchers as effective to incorporate the managerial flexibilities into the project value(Boer. 2003; Boyle. 1988; Jun and Kim. 2009). In infrastructure projects, design flexibility or staged construction may provide the management with chances of flexible decision-making, which causes the added cash flow or project value, against varying conditions showing new information as the market becomes more certain(Meredith and Mantel. 1995). Unfortunately, assessing this managerial flexibility costs lots of money and time but it still needs to be appropriately evaluated. However, the NPV analysis is limited to evaluate this added value that comes from the managerial flexibility. Equation<1> is the basic form of the NPV analysis discounting the future cash flows at required rate of return, WACC(Weighted Average Cost of Capital)(Brigham and Houston. 2004):

$$NPV = -I_0 + \sum_{i=1}^{t} \frac{FCF_i}{(1+WACC)^i} \qquad <1>$$

$$WACC = R_e \times \frac{E}{A} + R_d \times \frac{D}{A} \times (1-T) \qquad <2>$$

In Equation(1), I_0 is initial investment, i is time increment and FCF_i is free cash flow after tax at time t. WACC of the firm or project is defined in Equation<2>. In Equation<2>, E is equity investment, R_e is cost of equity, A is total capital investment, R_d is cost of debt, T is tax rate, and D is debt. In infrastructure projects, WACC is, in general, determined based on Equation<2> and the calculated WACC is applied in Equation<1> to find net present value of the project. WACC stands for a company's weighted average cost of capital reflecting cost of debt and cost of equity, and it is employed to evaluate projects matching a firm's existing operating assets and associated risks(Brigham and Houston. 2004; Dias and Ioannou. 1995). While the determination of R_d, T, D, E and A is relatively simple, the cost of equity of R_e, which is a measurement of the appropriate required return the equity investors should expect on against their equity investments, given the level of risk of such investments, can be estimated by CAPM(Capital Asset Pricing Model)(Sharpe. 1964; Trigeorgis. 1999). Equation<3> shows the way to estimate R_e based on the CAPM(Brigham and Houston. 2004).

$$R_e = R_f + \beta_e (R_m - R_f) \qquad (3)$$

Where, R_f is risk-free rate, β_e is risk measurement of equity investor, and R_m is rate of return on overall market. In infrastructure projects, since the risk premium due to uncertainties in the projects such as country or sector risk should be added to the cost of equity, actual risk-adjusted discount rate used in a real investment analysis can be greater than R_e(Jun. 2008).

2. Option Pricing Theory

The option pricing theory developed by Black, Scholes(1973), and Merton(1973), for pricing financial derivatives is the building block of the RCP option pricing approach in this paper. The concept of option pricing theory in finance was imported to seek to value managerial flexibilities on real assets or properties. The option pricing theory is based on the assumption that the stock price follows an uncertain diffusion process of a log-normal distribution called a 'Geometric Brownian Motion' proven to appropriately model the price of an asymmetric payoff of financial securities(Black and Scholes. 1973; Luenberger. 1998). The uncertainty of the value of real asset is reasonably reflected through this diffusion process(Brennan and Schwartz. 1978; Dixit and Pindyck. 1994). Equation<4> describes the diffusion process of Geometric Brownian motion process in a capital market:

$$\frac{dS}{S} = \mu \, dt + \sigma \, dz \qquad <4>$$

Where, S is stock price, μ is instantaneous rate of return, σ is instantaneous standard deviation of rate of return, and dz is random increment to a standard wiener process. In the option pricing theory, which falls into two folds; Black-Scholes model(continuous-time approach) and Binomial model(discrete-time approach), the value of a European call option can be obtained by solving the partial differential equation derived by Black and Scholes(1973), subject to one terminal and two boundary conditions(Jun and Kim. 2009). This Black-Scholes equation is based on the complex mathematical process and that is why the Black-Scholes equation is analytically limited in modeling and calculating the option. For this reason, it has been necessary to use a numerical solution such as the binomial model that this paper follows(Cox et al. 1979). Recently, we can see there have been more efforts to evaluate real assets based on the option pricing theory.

3. Asymmetric Payoff: Combination of Uncertainty and Managerial Flexibility

Even if the NPV analysis is considered appropriate to evaluate the project in light of being consistent with the firm's objective of maximizing the shareholders' utilities, when the uncertainty is involved in an investment analysis, the discount rate used in this method should be fairly modified against the related risks based on the CAPM(Copeland and Weston. 1988; Jun and Kim. 2009). However, there exits problematic argument that the NPV analysis can not capture the characteristics of the managerial flexibilities, during the uncertain behavior of the project value or cash flow, when the future is unlike expected. The managerial flexibilities provide specific kinds of asymmetric payoffs analogous to those of the derivatives in a financial market(Cox et al. 1979). Therefore, the option pricing theory can be effective to price such complicated contingencies of the managerial flexibilities. Followings are examples of the asymmetric payoffs in financial call and put options(Hull and White. 1990):

$$F_{call}(S_t,\ t) = Max\,[\,0,\ S_t - X\,] \qquad <5>$$

$$F_{put}(S_t,\ t) = Max\,[\,0,\ X - S_t\,] \qquad <6>$$

Where, F_{call} is call option value, F_{put} is put option value, S_t is stock price at time t, and X is exercise price.

4. RCP Agreement as a Contingent Claim

In a BOT toll road system, initial traffic volume and annual traffic volume growth rate are most important risky variables since these directly affect the project revenue(Cheah and Liu. 2006; Jun. 2008). Unfortunately, as it may be hardly possible to exactly predict the future change of these two revenue(or cash flow) components being likely to happen in a real project, there should be an alternation to prevent the government from being exploited in its revenue than they have to endure against the BOT developer when the project revenue is mistakenly estimated. If the traffic volume and the traffic volume growth rate are excessively surpassing the initially pre-determined level agreed on in the negotiation process between the government and the BOT developer, in turn, the BOT developer can enjoy the surplus way over it must deserve, the government should have a way to ask claim for excessive benefit paid to the BOT developer so that the government quit or mitigate kinds of averse situations due to the revenue exploitation. This practice is called RCP agreement that stipulates the payment from the BOT developer to the government(Jun. 2008). That is, the RCP can be thought as a ceiling on the return of the BOT developer and the repaid capital to the government can be redistributed for the purpose of financing the related facilities, increasing the taxes, reducing toll rates to benefit the users, or participating in the project as a direct equity investor. On the other hand, if the realized revenue is lower than initially projected, the government have to put itself in a position of subsidizing revenue shortfall as already agreed in a form of contract. This agreement, which is called “MRG(Minimum Revenue Guarantee)”, helps the government attract the BOT developer and long-term financial investors at the stage of fundraising(Jun and Sharkawy. 2008). Generally, unsolicited proposals from the private developer in projects often occur and inappropriate level of the RCP agreement may put the government in a side of revenue exploitation with an unfair deal proposed by the private developer(Cheah and Liu. 2006; Hodges. 2003). Likewise, because failure to consider the RCP value tends to look at the project mistakenly positive, this contingency also needs to be carefully considered from the BOT developer's point of view as well.

The RCP agreement can be formulated in a form of financial call option, which shows us the obvious asymmetric payoff condition, described in Equation<7>. The underlying concept that the RCP agreement entails is, during the concession, if the realized cash flow in each year i surpasses way over negotiated level of the projected cash flow which is already signed in a contract the government can ask claim for the revenue surplus to the BOT developer.

$$RCF_i = Max\begin{bmatrix} Realized\ FCF_e\ \ at\ \ year\ \ i \\ \quad - Projected\ \ FCF_e\ \ at\ \ year\ \ i,\ 0 \end{bmatrix} = Max\ [FCF_{ir} - FCF_{ip},\ 0]$$ <7>

Where, RCF_i is RCP value at year i, FCF_e is free cash flow on equity, FCF_{ir} is realized cash flow at year i, and FCF_{ip} is projected cash flow at year i. As an RCP value, the BOT developer's obligation to pay at year i, RCF_i, relies on the relative value between realized cash flow at year i, FCF_{ir}, and projected cash flow at year i, FCF_{ip}, as shown in Equation<7>(Cheah and Liu. 2006).

Finally, the total value of RCP agreement until the end of concession, year n, can be obtained by summing up the discounted RCF_i at risk free rate to time “0”. Equation<8> show this calculation process:

$$RCP \;=\; \sum_{i=1}^{n} \frac{ECF_i}{(1+R_f)^i} \qquad <8>$$

When necessary to put the RCP agreement in a BOT project, the way for both of the government and the BOT developer to specify the level of capped revenue can be based on the IRR(Internal Rate of Return), revenue, or traffic volume. And, this level would be fixed through the negotiation process between the government and the BOT developer.

III. RCP Option Pricing Approach

This paper takes into account a development of an option pricing framework based on the binomial model(Cox et al. 1979), which is intuitive to identify the complexly-combined contingent claims, in quantitatively evaluating the RCP agreement in a BOT project. The underlying asset, the BOT project value, is assumed to follow a specific diffusion process called “Geometric Brownian Motion” process reflecting the uncertain dynamics of the project value change over time. The option modeling process keeps following disciplines.

- The RCP agreement is numerically modeled as repeatedly-exercisable call option along with the appropriate financial and mathematical process
- The impact of the RCP agreement on the project value is considered at the level of equity to look at the project from the BOT developer and the government's points of views
- The developing process of the RCP model is limited to three-time step for space and mathematical complexity

Finally, the numerically-modeled RCP option approach is tested to show enough applicability through the hypothetical case of the BOT toll system. Followings are the processes to build the RCP option model and the hypothetical BOT toll case.

1. Underlying Asset and Its Dynamics

The first modeling procedure of RCP agreement is to choose the underlying asset and its dynamics.

The value change of the underlying asset, BOT project, affects the RCP value because the option value varies contingent on the underlying asset over time. As the debt payment against the financial investors depends on the future cash flows as collateral in a project finance, the forecasted cash flow, which affects the project value, becomes a main source of the underlying asset, in turn, the uncertainty of the project value would be risky variable during the concession(Beidleman et al. 1990; Finnerty. 1996). Afterward, the project value is assumed to fluctuate over time, due to the dynamic change of market conditions until the end of the concession, with a specific diffusion process, so called "Geometric Brownian Motion" process in Equation<9>(Dixit and Pindyck. 1994). This notion helps easily assume a structure for the dynamics and uncertainties of the underlying risky asset 'project value.'

$$\frac{dV}{V} = \mu\, dt + \sigma\, dz \qquad <9>$$

Where, V is market value of a completed project, μ is market required rate of return by project, and σ is volatility of rate of return by project.

2. Initial Project Value

To reflect the dynamics of the project value, it is necessary to have the initial project value identical to the discounted sum of the future cash flows, aside form the initial investment cost, to the present with appropriate risk-adjusted discount rate. If considers the BOT developer and the government's points of views, the dynamics of the project value on equity can be shown in Equation<10>. FCF_{ei} in Equation<10> is free cash flow on equity at year i.

$$V_I = \sum_{i=1}^{n} \frac{FCF_{ei}}{(1+R_e)^i} \qquad <10>$$

3. Volatility

Volatility, σ, defined as a standard deviation of rate of return in cash flow return is a measurement of the 'risk' in finance and considered as an important factor to dominate the option value(Brigham and Houston. 2004; Hull. 1997). There are some ways; Logarithmic cash flow return approach, Monte Carlo simulation, Implied volatility, and so on, to find this value and, according to the extent required for each financial analysis; level of the accuracy or the convenience, the financial modeler chooses the method believed proper. Amongst various methodologies to find the volatility, because the logarithmic cash flow return approach is considered easy to be simply applied in a financial analysis, it is widely used in the valuation of real assets in many industries(Jun. 2008). The basic data used to calculate the volatility may be historic or future estimates of cash flow returns agreed between the public and the private in a project.

1) Up/Down Movements and Risk Neutral Probabilities

In the project value change over time, “up” and “down” movements, u and d, which are multiplied with the initial project value V_I to reflect the uncertain behavior of the project value, are obtained with σ based on Equation<11> and <12>. By imposing $u=1/d$ for convenience, the up and down movements and risk neutral probabilities can be obtained from Equation<11> to <14>(Cox et al. 1979). When we say n is number of times that one period of time is divided and $\triangle t$ is time interval, n multiplied by $\triangle t$ equals to 1.

$$u=\mathrm{Exp}[\sigma\sqrt{\triangle t}] \qquad \text{<11>}$$

$$d=\mathrm{Exp}[-\sigma\sqrt{\triangle t}] \qquad \text{<12>}$$

Where, u and d are up and down movement multipliers of project value respectively and R is multiplier of risk−free rate, $\mathrm{Exp}[r\triangle t]$ while continuous time approach. However, it can be considered to obtain u and d under the assumptions that the risk−neutral probabilities q and $1-q$ are equal to 0.5 for the convenience of the calculation(Hull. 1997). By keeping the risk−neutral probabilities stable, it becomes trivial whether the binomial tree is huge and the calculation is heavy. Equation<13> and <14> are the replacements of Equation<11> and <12> while imposing $q=1-q=0.5$.

$$u=\mathrm{Exp}\left[\left(r-\frac{1}{2}\sigma^2\right)\Delta t+\sigma\sqrt{\Delta t}\right] \qquad \text{<13>}$$

$$d=\mathrm{Exp}\left[\left(r-\frac{1}{2}\sigma^2\right)\Delta t-\sigma\sqrt{\Delta t}\right] \qquad \text{<14>}$$

Afterward, we can find the risk−neutral probabilities, q and $1-q$. The financial implication of risk−neutral probabilities is that the world where the project is being implemented is risk−neutral so that the financial modeler does not have to waste their time to find any arbitrary risk−adjusted discount rate. The notion of this “risk−neutral world” that the option pricing theory is based on in financial economics is tremendously helpful while the use of the financial engineering technique in evaluating complex financial products such as option, future, derivatives(Copeland and Antikarov. 2001). The risk-neutral probabilities are shown in Equation<15> and <16>.

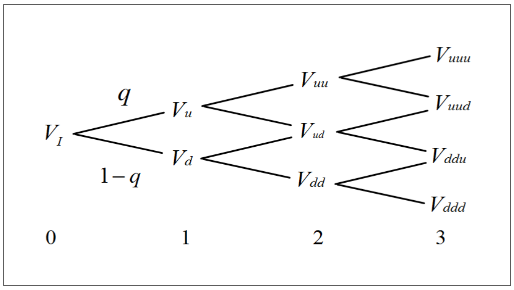
$$q = \frac{R-d}{u-d} \quad (15) \quad \text{and} \quad 1-q = \frac{u-R}{u-d}$$ <16>

2) Binomial Tree with an Underlying Asset

This is time to build a binomial tree, with V_I, σ, u, and d taken from above, that stands for all likely project values considering the uncertainties over time and the realized project value.

The followings are likely project values over time.

Figure 1_Binomial Tree of Underlying Asset, Project Value



At t=1

$$V_u = u \cdot V_I = \text{Exp}\left[1 \cdot \left(\left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}\right)\right] \cdot V_I$$ <17>

$$V_d = d \cdot V_I = \text{Exp}\left[1 \cdot \left(\left(r - \frac{1}{2}\sigma^2\right)\Delta t - \sigma\sqrt{\Delta t}\right)\right] \cdot V_I$$ <18>

At t=2

$$V_{uu} = u^2 \cdot V_I = \text{Exp}\left[2 \cdot \left(\left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}\right)\right] \cdot V_I$$ <19>

$$V_{ud} = u \cdot d \cdot V_I = \text{Exp}\left[\begin{array}{l}\left[1 \cdot \left(\left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}\right)\right] \\ + \left[1 \cdot \left(\left(r - \frac{1}{2}\sigma^2\right)\Delta t - \sigma\sqrt{\Delta t}\right)\right]\end{array}\right] \cdot V_I$$ <20>

$$V_{dd} = d^2 \cdot V_I = \text{Exp}\left[2 \cdot \left(\left(r - \frac{1}{2}\sigma^2\right)\Delta t - \sigma\sqrt{\Delta t}\right)\right] \cdot V_I$$ <21>

At t=3

$$V_{uuu} = u^3 \cdot V_I = \text{Exp}\left[3 \cdot \left(\left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}\right)\right] \cdot V_I$$ <22>

$$V_{uud} = u^2 \cdot d \cdot V_I$$

$$= \mathrm{Exp}\left[\begin{array}{l}\left[2\cdot\left(\left(r-\frac{1}{2}\sigma^2\right)\triangle t+\sigma\sqrt{\triangle t}\right)\right] \\ \quad +\left[1\cdot\left(\left(r-\frac{1}{2}\sigma^2\right)\triangle t-\sigma\sqrt{\triangle t}\right)\right]\end{array}\right]\cdot V_I \qquad \text{<23>}$$

$$\begin{aligned} V_{ddu} &= d^2\cdot u\cdot V_I \\ &= \mathrm{Exp}\left[\begin{array}{l}\left[2\cdot\left(\left(r-\frac{1}{2}\sigma^2\right)\triangle t-\sigma\sqrt{\triangle t}\right)\right] \\ \quad +\left[1\cdot\left(\left(r-\frac{1}{2}\sigma^2\right)\triangle t+\sigma\sqrt{\triangle t}\right)\right]\end{array}\right]\cdot V_I \end{aligned} \qquad \text{<24>}$$

$$V_{ddd} = d^3\cdot V_I = \mathrm{Exp}\left[3\cdot\left(\left(r-\frac{1}{2}\sigma^2\right)\triangle t-\sigma\sqrt{\triangle t}\right)\right]\cdot V_I \qquad \text{<25>}$$

4. RCP Agreement as a Call Option

1) Asymmetric Payoff Condition

The RCP agreement can be numerically formulated as a financial call option based on the asymmetric payoff shown in Equation<7>. However, the option considered in this paper is modeled at the level of project value but the level of cash flow as the volatility we need based on the project value is not exactly identical to that by the cash flow. The asymmetric payoff generated from the RCP agreement during the concession is as below. Where, RCP_i is the RCP value at year i.

$$RCP_i = Max\left[\begin{array}{l} Realized\ Project\ value\ on\ Equity \\ \ at\ year\ i \\ -\ Projected\ Project\ Value\ on\ Equity \\ \quad at\ year\ i \qquad\qquad\qquad , \quad 0 \end{array}\right] \qquad \text{<26>}$$

Recall that, in the option pricing theory the project value change follows a geometric Brownian motion process contingent on two major factors: the term of the stable rate of return which excludes the possibility of uncertain price behavior and that of the uncertain rate of return which follows random selection process during the operation(Hull and White. 1990). Here, the varying project value with the stable rate of return is assumed as a realized project value “exercise price” as follows. X_n represents an exercise price at time t.

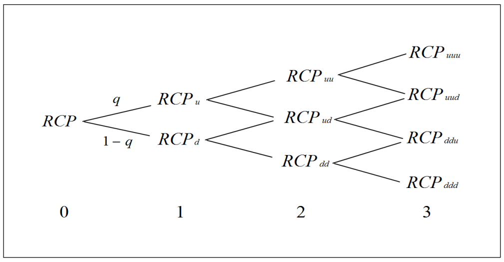
At t=n

$$X_n = \mathrm{Exp}\left[n\cdot\left(r-\frac{1}{2}\sigma^2\right)\triangle t\right]\cdot V_I \qquad <27>$$

2) Repeatedly-exercisable RCP Call Option

Throughout the above steps, we finally can construct the asymmetric payoff conditions for every node at each time step as shown in Figure 2 and Equation<28> to <36>.

Figure 2 _ RCP Option Value in Binomial Tree



At t=1

$$RCP_u = Max[\,V_u - X_1,\ 0\,] \qquad <28>$$

$$RCP_d = Max[\,V_d - X_1,\ 0\,] \qquad <29>$$

At t=2

$$RCP_{uu} = Max[\,V_{uu} - X_2,\ 0\,] \qquad <30>$$

$$RCP_{ud} = Max\ [\,V_{ud} - X_2,\ 0\,] \qquad <31>$$

$$RCP_{dd} = Max\ [\ V_{dd} - X_2,\ 0\,] \qquad <32>$$

At t=3

$$RCP_{uuu} = Max[\,V_{uuu} - X_3,\ 0\,] \qquad <33>$$

$$RCP_{uud} = Max[\,V_{uud} - X_3,\ 0\,] \qquad <34>$$

$$RCP_{ddu} = Max[\,V_{ddu} - X_3,\ 0\,] \qquad <35>$$

$$RCP_{ddd} = Max\ [\ V_{ddd} - X_3,\ 0\,] \qquad <36>$$

5. RCP Option Calculation

As the final step of modeling the RCP option, the calculation of option value begins from the end of the binomial tree backward recursively in Figure 2. The selected option value based on the asymmetric payoff condition at every node is obtained by being calculated with the parameters of q, $1-q$, and R.

For instance, while finding the option value at node 'uu', RCP_{uu}, in Figure 2, we can expect two events of whether an RCP option is exercised or not. As we should selectively choose the maximized option value at every node under the assumption that the reasonable investors do their best to maximize profits, the larger option value regardless of option's exercise will be selected(Jun. 2008). Hence, the only one option value should be chosen between when exercised and when not exercised whichever is larger. Equation<37> and <38> show these two possible events at node "uu". And, the larger one is captured as an RCP value in Equation<39>. This process is sequentially and backward recursively iterated at every node from the end of the binomial tree to the present.

$$RCP_{uu(Exercised)} = Max\left[\, V_{uu} - X_2,\ 0\right] \qquad \text{<37>}$$

$$RCP_{uu(Not\ Exercised)} = \frac{\begin{bmatrix} q \cdot Max\left[V_{uuu} - X_3,\ 0\right] \\ \qquad + (1-q) \cdot Max\left[V_{uud} - X_3,\ 0\right] \end{bmatrix}}{\text{Exp}\left[r \triangle t\right]} \qquad \text{<38>}$$

$$RCP_{uu} = Max\begin{bmatrix} \dfrac{\begin{bmatrix} q \cdot Max\left[V_{uuu} - X_3,\ 0\right] \\ \qquad + (1-q) \cdot Max\left[V_{uud} - X_3, 0\right] \end{bmatrix}}{\text{Exp}\left[r \triangle t\right]} \\ , Max\left[V_{uu} - X_2,\ 0\right] \end{bmatrix} \qquad \text{<39>}$$

Equation<40> to <45> describes all likely asymmetric payoff conditions and RCP option values at all nodes in the three-step binomial tree.

At t=2

$$RCP_{uu} = Max\left[\frac{\left[q \cdot RCP_{uuu} + (1-q) \cdot RCP_{uud}\right]}{\text{Exp}\left[r \triangle t\right]},\ \ V_{uu} - X_2\right] \qquad \text{<40>}$$

$$RCP_{ud} = Max\left[\frac{\left[q \cdot RCP_{udu} + (1-q) \cdot RCP_{udd}\right]}{\text{Exp}\left[r \triangle t\right]},\ \ V_{ud} - X_2\right] \qquad \text{<41>}$$

$$RCP_{dd} = Max\left[\frac{\left[q \cdot RCP_{ddu} + (1-q) \cdot RCP_{ddd}\right]}{\text{Exp}\left[r \triangle t\right]},\ \ V_{dd} - X_2\right] \qquad \text{<42>}$$

At t=1

$$RCP_u = Max\left[\frac{[q\cdot RCP_{uu} + (1-q)\cdot RCP_{ud}]}{\text{Exp}[r\triangle t]},\quad V_u - X_1\right] = Max\left[\frac{\left[q\cdot Max[V_{uu}-X_2, 0] + (1-q)\cdot Max[V_{ud}-X_2,\ 0]\right]}{\text{Exp}[r\triangle t]}, V_u - X_1\right] \quad <43>$$

$$RCP_d = Max\left[\frac{[q\cdot RCP_{du} + (1-q)\cdot RCP_{dd}]}{\text{Exp}[r\triangle t]},\quad V_d - X_1\right] = Max\left[\frac{\left[q\cdot Max[V_{du}-X_2, 0] + (1-q)\cdot Max[V_{dd}-X_2,\ 0]\right]}{\text{Exp}[r\triangle t]}, V_d - X_1\right] \quad <44>$$

At t=0

$$RCP = \frac{q\cdot RCP_u + (1-q)\cdot RCP_d}{\text{Exp}[r\triangle t]} = \frac{\left[qMax\left[\frac{[qRCP_{uu}+(1-q)RCP_{ud}]}{\text{Exp}[r\triangle t]}, V_u - X_1\right] + (1-q)Max\left[\frac{[qRCP_{du}+(1-q)RCP_{dd}]}{\text{Exp}[r\triangle t]}, V_d - X_1\right]\right]}{\text{Exp}[r\triangle t]} \quad <45>$$

IV. Hypothetical Case Study

This chapter is to apply the RCP option pricing framework into the hypothetical case of the BOT toll road, which is simplified version of a real project, to show the applicability of the approach. Table 1 describes the basic data and information of the BOT case example where the capital expenditure, operating expenditure, and average toll rate are assumed to increase at a annual level of 3%. Figure 3 describes the expected cash flow model given data and information in Table 1.

1. NPV Analysis

With market risk premium, $R_m - R_f$ of 5.1%, and β, which is the weighted average of the equity investors' β, we finally find NPV_e equal to \$2.02 million without considering the RCP agreement.

Table 1_Hypothetical BOT Toll Road Case

Capital Structure	
Project Construction Cost	$143.47 M(3 years)
Debt : Equity(71.8:28.2)	$103.01 M : $ 40.46 M
Debt	Senior: 13 years(7.25 %)
Capital Expenditure	$2.61 M(Every 5 years)
Operating Expenditure	$2.59 M(Every year)
Corporate Tax Rate	27.5%
Initial Traffic Volume (Mean / Volatility)	7.153 / 1.94 M(Year)
Traffic Volume Growth Rate(Mean / Volatility)	2.34 / 0.79 %
Average Toll Rate	$ 1.87
Concession Period	30 Years(From 2005)
RCP Agreement	110% of Expected Revenue (30 years)
Market Rate of Return	10.4%
Risk Free Rate	5.3%
Cost of Equity	12.848%
Average β	1.48
Volatility	0.083

Figure 3_Cash Flow Model of BOT Toll Road System

(M: Million, $: Dollar)

Year	2002	2003	2004	2005	2006	……	2032	2033	2034
Traffic Volume(M) Toll Rate($)				7.153 1.87	7.398 1.93	……	13.38 4.16	13.67 4.29	13.96 4.42
Gross Revenue(M, $)				13.40	14.28	……	55.69	58.59	61.65
CAPEX(M, $)	40.46					……			5.96
OPEX(M, $)				2.50	2.58	……	5.55	5.72	5.89
EBIT(M, $)	−40.46			10.90	11.70	……	50.14	52.87	49.80
Debt Service(M, $)						……			
Taxes(M, $)				3.00	3.22	……	13.79	14.54	13.69
FCF on Equity(M, $)	−40.46	0.00	0.00	7.90	8.48	……	36.35	38.33	36.10

Equation<46> and <47> show the way R_e and NPV_e are obtained.

$$R_e = R_f + \beta \times (R_m - R_f) = 5.3\% + (5.1 \times 1.48) = 12.848\%$$ <46>

$$NPV_e = \frac{-40.46}{(1.1285)^0} + \frac{0}{(1.1285)^1} + \frac{0}{(1.1285)^2} + ... + \frac{38.33}{(1.1285)^{31}} + \frac{36.10}{(1.1285)^{32}} = \$2.02\,Million$$ <47>

2. RCP Option Pricing Approach

We can have some calculated parameters of V_I, u, d, q, and $1-q$, based on the steps described earlier, necessary to be ready for the option pricing approach and building a binomial tree to render all the likely project values supposed to happen during the concession. The initial project value V_I is as Equation<48>.

$$V_I = \sum_{i=1}^{n} \frac{FCF_{ei}}{(1+R_e)^i} = \frac{7.90}{(1.1285)^1} + \frac{8.48}{(1.1285)^2} + ... + \frac{38.33}{(1.1285)^{29}} + \frac{36.10}{(1.1285)^{30}} = \$54.10\,Million$$ <48>

u is obtained by $\mathrm{Exp}[((0.053-(1/2)(0.083)^2)\times 1) + (0.083\sqrt{1})]$ and is 1.142. d also can be obtained by the formula of $\mathrm{Exp}[((0.053-(1/2)(0.083)^2)\times 1) - (0.083\sqrt{1})]$. Finally, d is 0.967. When it comes to the exercise price dependent on the projected project value, since the projected project value at time '0' is the same as the initial project value multiplied by 1.1 according to the agreement the revenue surplus which is way over 110% of the expected cash flow will be paid back to the government, the initial exercise price is 1.1×54.10 identical to 59.51 at the first year of the operation. Then, this value increases at annual rate of $r-(1/2)\sigma^2 = 0.053-(1/2)\times 0.083^2 = 0.05$ over time from second year of the concession. Table 2 describes the parameters needed to start calculating the RCP option value on the basis of discrete−time approach. The RCP value is obtained from backward recursive calculation process regardless of whether or not the option is exercised at every node in the binomial tree by being

Table 2_ Calculated Parameters for Option Pricing Approach

	Calculated Parameters
V_I	\$54.10 Million
u	1.142
d	0.967
$q = 1-q$	0.5
Initial Exercise Price(X_1)	\$59.51 Million

Figure 4 _ RCP Option Value of BOT Toll Road System (Million Dollars)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
15.86	22	29	38	49	62	77	95	116	140	168	136	164	196	233	276
	12	17	23	31	41	53	68	85	104	127	101	124	150	180	216
		8	12	17	24	33	45	58	74	93	72	90	111	136	165
			6	8	12	18	26	36	49	64	48	62	78	98	122
				3	5	8	12	18	27	39	30	40	52	67	86
					2	3	5	7	11	18	17	24	32	43	56
						1	2	3	4	6	9	12	18	25	34
							1	1	2	3	4	6	9	13	18
								0	1	1	1	2	4	6	8
									0	0	0	1	1	2	3
										0	0	0	0	1	1
											0	0	0	0	0
												0	0	0	0
													0	0	0
														0	0
															0

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
326	383	449	524	612	712	827	959	1111	1285	1485	1714	1977	2278	2622
257	304	359	422	494	578	674	785	912	1058	1225	1418	1638	1891	2181
198	237	283	335	395	465	545	637	743	865	1005	1167	1352	1564	1807
149	181	218	261	311	369	435	512	600	702	819	954	1109	1286	1491
107	133	163	199	240	287	342	406	479	564	661	774	903	1051	1222
73	93	117	146	179	218	264	316	377	447	528	621	729	852	995
46	61	79	102	128	160	197	240	290	347	414	492	581	684	803
26	36	49	65	86	111	140	175	216	263	318	382	456	541	640
13	19	27	38	52	70	93	121	154	192	237	289	350	420	502
5	8	12	19	27	39	55	76	101	132	168	211	260	318	385
2	3	5	8	12	18	28	41	59	81	110	144	184	231	285
0	1	1	2	4	7	11	18	28	42	62	88	120	157	202
0	0	0	1	1	2	3	6	10	16	27	43	65	95	130
0	0	0	0	0	0	1	1	2	4	7	14	24	42	70
0	0	0	0	0	0	0	0	0	0	1	2	4	9	19
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0
				0	0	0	0	0	0	0	0	0	0	0
					0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0
							0	0	0	0	0	0	0	0
								0	0	0	0	0	0	0
									0	0	0	0	0	0
										0	0	0	0	0
											0	0	0	0
												0	0	0
													0	0
														0

discounted at risk-free rate with the consideration of risk neutral probabilities. As a result, the calculation gives us the RCP value, in 2007, $15.86 million as described in Figure 4.

3. Results

Upon discounting the RCP value of $15.86 million(in 2004) at risk-free rate, we found the RCP value as of 2002 is $14.26 million. This approximately accounts for 706% of NPV_e, $2.02 million, and 35.7% of the initial equity investment, $40 million, respectively. Note that NPV_e under the RCP agreement, $16.28 million, accounts for 114.17% of NPV_e not considering the RCP agreement, $14.26 million. This result indicates the RCP value can not be overlooked because of its huge impact on the BOT project value. For the sensitivity of the RCP value against the changes of two revenue components; the initial traffic volume and the traffic volume growth rate, Figure 5 and 6 afford as both of two revenue components increase the RCP value increases. In the option pricing approach, the revenue components which consist of the underlying asset are proven to be important for the management to better prepare for and react against operating risk due to the fluctuating revenue because they directly affect the underlying

Figure 5 _ RCP Value against the Change of Initial Traffic Volume

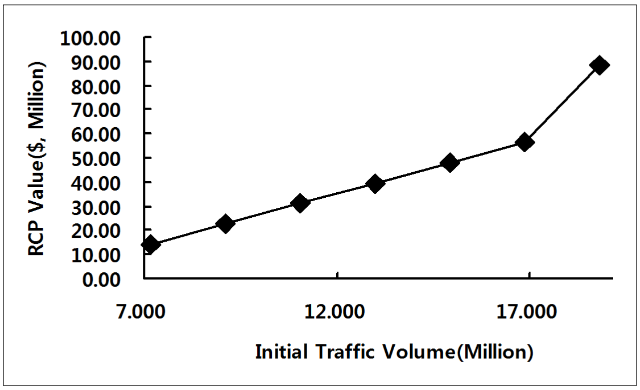
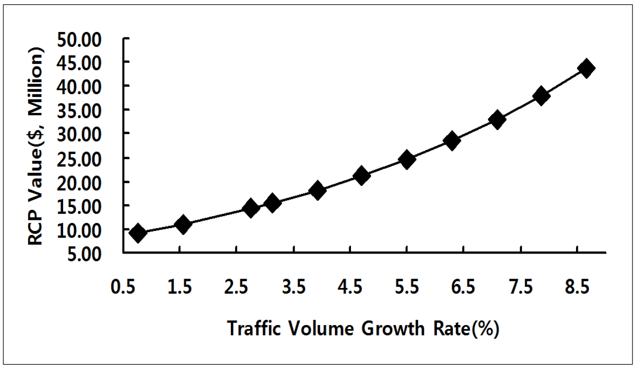


Figure 6_ RCP Value against the Change of Traffic Volume Growth Rate



asset value(initial project value in this paper) that the option value depends on. More specifically, since the volatility of each revenue component is the most important input to vary the option value, the management should put more time and effort in better predicting the volatility or fluctuation of the initial traffic volume and the traffic volume growth rate. Note that, when the option pricing theory is applied in evaluating the managerial flexibilities of impending toll projects, the fluctuation, called "volatility", of the project value or revenue components usually can be estimated based on the historical cash flows of the existing toll project under the similar condition. However, since there may hardly exist the BOT projects involving lots of similarities that we can refer to during the financial modeling, the expected cash flow of the BOT project under consideration may be available upon careful adjustment.

Figure 7 and 8 and Table 4 describe the sensitivity of the RCP value to the change of the volatilities(standard deviations) of two revenue components. The result renders while the RCP value depends on the volatility of the initial traffic volume, the volatility of the traffic volume growth rate does not seem to impact on the RCP value. This implies that, for the BOT developer to reasonably decide the revenue ceiling during the bidding process in this BOT case, the BOT developer has to more focus on predicting the probability distribution of the initial traffic volume rather than that of the traffic volume growth rate, the impact of which may be ignorable, since

Table 3_ Volatility of the BOT Project Value against the Volatility Change of Two Revenue Components

	Volatility of Traffic Volume Growth Rate(σ_{GR})				
Volatility of Initial Traffic Volume(σ_{TV})		0.40	0.79	1.19	1.58
	0.97	0.042	0.041	0.041	0.041
	1.94	0.080	0.083	0.080	0.081
	2.91	0.117	0.117	0.118	0.117
	3.88	0.152	0.150	0.152	0.152

the probability distribution of the initial traffic volume significantly affects the RCP value that is cost to the BOT developer. Furthermore, it is no doubt the government should keep in mind the importance of the RCP level either in order not to be exploited from the lopsided deal unfairly proposed by the BOT developer. However, another point needs to be better emphasized is that the RCP option value against the volatility of the BOT project value(underlying asset) behaves the same way as the financial option in which the option value increases while the volatility of the underlying asset increases. Importantly, during the evaluation of the managerial flexibilities, the real asset projects like BOT toll road systems can not be dealt with in a same manner and must be individually customized for the condition that the project is subject to because each project has unique characteristics(probability distributions of initial traffic volume and traffic volume growth rate, maximum capacity, time value of money, and so on) which all interact one another in a complex manner(Cheah and Liu. 2006). As for the effect of the revenue ceiling level, the increasing revenue ceiling seems to reduce the RCP value as the probability of the threshold to exercise the RCP call option becomes lower(Figure 9). Hence, if the government or the BOT developer want to identify the appropriate range of the MRG

Figure 7_Sensitivity of the RCP Value to σ_{TV}(σ_{GR} =0.79)

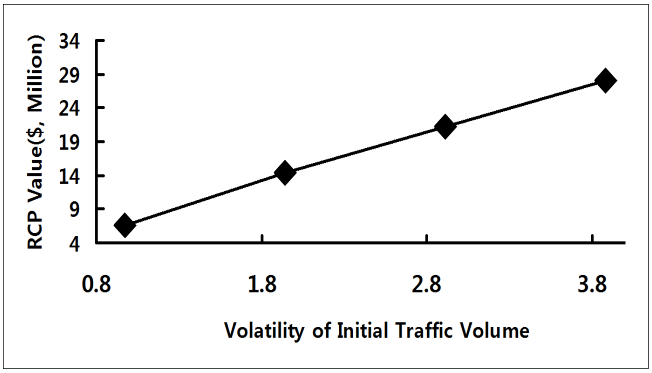


Figure 8_Sensitivity of the RCP Value to σ_{GR}(σ_{TV} =1.94)

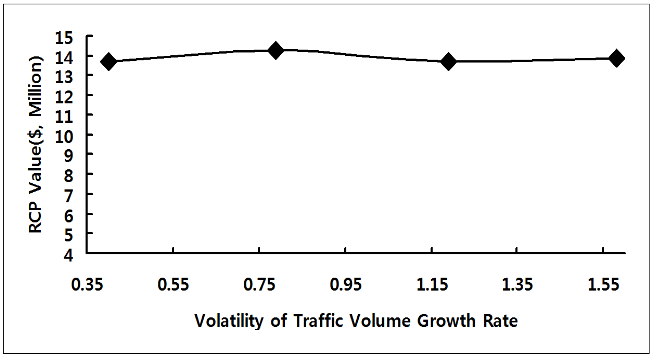
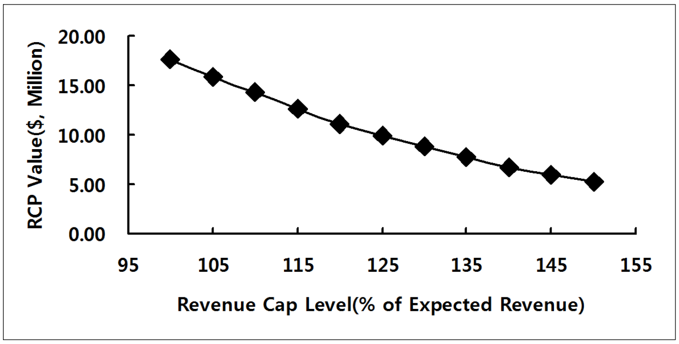


Table 4_RCP Value over the Volatility Change of Two Revenue Components

	Volatility of Traffic Volume Growth Rate(σ_{GR})				
Volatility of Initial Traffic Volume(σ_{TV})		0.40	0.79	1.19	1.58
	0.97	6.75	6.58	6.58	6.58
	1.94	13.68	14.26	13.68	13.88
	2.91	21.11	21.11	21.32	21.11
	3.88	28.59	28.15	28.59	28.59

Figure 9 _ RCP Value to the Change of Revenue Cap Level



agreement balanced with the RCP agreement level in a BOT project, it is possible by matching the MRG agreement value, which may be obtained through the appropriate option pricing modelling process, with the RCP agreement value we can calculate in a way described in this paper.

V. Conclusions

Because the RCP agreement is cost to the BOT developer and value to the government, it is important to understand the impact of the RCP agreement on the financial feasibility. In light of this, some meaningful conclusions drawn from this paper are as follows. First, the RCP value taken from the option pricing approach turns out to have significant impact on the net present value on equity in BOT toll road case. Therefore, the negotiation process associated with the RCP agreement should be better considered. Moreover, because the projected project value used as an exercise price in this paper is the only controllable parameter and heavily relies on the revenue ceiling level, the process to decide the revenue cap level should be carefully taken into account. Second, during the bidding process, regardless of the national economy condition, the government seeks to lower the revenue ceiling and the MRG level while the BOT developer makes an effort to raise the RCP and the MRG level within reasonably-accepted range. However, from the social point of view, the level of the RCP and MRG agreements during the investment analysis needs to be fairly considered and reflected in the bidding process by keeping in mind that the MRG and RCP cost the government and the BOT developer respectively should things go wrong. To do so, the option pricing approach can be an efficient alternation to suggest the fairly quantified RCP or MRG agreement level. Third, the option approach, based on the binomial model, formulated in this paper is relatively easy to use in a real BOT project rather than the complex analytical mathematics, the Black-Scholes equation, thanks to its derivation from simple algebra level of the NPV analysis. In addition, the way to formulate the RCP agreement may be applied to evaluate other managerial flexibilities taking place in a BOT project through the adequate modification process.

In this paper, the option pricing approach applied provides a quantitative way which seems to have a flexible use in the real world to evaluate the RCP agreement value in a BOT project. However, there are still some open issues that mainly arise from the characteristics of the BOT project itself or the option pricing theory during the valuation process and it is supposed to further investigations concerning the following issues. First, the only reasonable financial modeling for a BOT project can justify the credibility of the option value. No matter what the purpose is, while the project revenue is intentionally distorted at any reason, the over or underestimated project revenue makes the option value go far beyond the acceptable level, in turn, changes the result of the negotiation. So, the accuracy in a use of the option pricing approach applicable in real assets can be justified as long as the estimated project revenue is close to the reality. To do so, further empirical studies are essential to improve the fundamental and validity of the option pricing frame in the BOT project world. In addition, the efforts on the parameter calibration for the BOT option pricing framework is necessary because the degree of accuracy in the parameters affects the preciseness of the option value. The consideration of the human behaviors with the game theory combined with the option pricing theory can be also thought as a meaningful effort to better reflect the reality of the BOT bidding process. Second, for both of the government and the BOT developer to identify the admissible level of the revenue ceiling considering the risk-reward concept, it needs to investigate how balanced the RCP agreement value is with that of the MRG level. To do so, it requires further research to understand the quantitative value of the MRG agreement. Even if the estimation of this MRG value, if possible to account for the option pricing approach, asks for almost corresponding amount of efforts as made in the valuation of the RCP agreement, it seems meaningful for either the government or the BOT developer to stand on a advantageous side of the bidding process. Finally, in a real world, since there are more various and complicated managerial flexibilities able to be formed as asymmetric payoffs, effort to identify, formulate, and evaluate these contingencies is more required.

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ABSTRACT

Appraisal of Revenue Cap Agreement in BOT Project Finance

Keywords: BOT Project Finance, Option Pricing Theory, NPV Analysis, Managerial Flexibility, Revenue Cap Agreement, Repeatedly-exercisable Call Option

Limited public funds for infrastructures have the government consider joining the private in a BOT project finance scheme. Generally, the BOT projects entail lots of managerial flexibilities that may induce the radical change of project's cash flows, an asymmetric payoff, when facing on the uncertainties comes from the BOT project finance's unique characteristics. Among various managerial flexibilities occur in the BOT projects, the revenue cap agreement is frequently used to protect the government from the private's revenue exploitation. However, its impact on the project value is not well understood because the popular capital budgeting theory, NPV(Net Present Value) analysis, is limited to assess the contingency of this agreement. The purpose of this paper is to develop the numerical model to better assess the impact of the revenue cap agreement on the project value with the concept of the option pricing theory and to suggest a theoretical framework in quantitatively evaluating this agreement. The approach applied in this paper is justified with the hypothetical BOT toll case to show its applicability and some meaningful conclusions are drawn from. The results by the option pricing concept are scrutinized over those by NPV analysis and, finally, the revenue cap value appears significant relative to the project value.

BOT 프로젝트 파이낸스에서 운영수익상한조항의 가치평가에 관한 연구

주제어: BOT 프로젝트 파이낸스, 옵션가격결정이론, NPV법, 경영상 유연성, 운영수익 상한조항, 반복행사가 가능한 콜옵션

일반적으로, BOT 프로젝트 파이낸스는 사업 특성상 여러 종류의 불확실성들을 수반하며 이러한 불확실성들은 경영상의 유연성들과 결합하면서 ‘비대칭수익구조’로 표현되는 현금흐름의 극심한 변화를 일으키게 된다. BOT 프로젝트 파이낸스에서 발생하는 여러 형태의 경영상 유연성 중, 운영수익상한조항은 민간사업자의 과다한 수익으로부터 정부를 보호하기 위해 자주 이용되는 방법이지만 흔히 사용되는 자본예산이론인 NPV(Net Present Value)법이 운영수익상한조항의 조건부청구특성에 의한 비대칭수익구조를 평가하기에 부적절하기 때문에 지금까지 이 조항이 사업가치에 미치는 정량적인 효과에 대해서는 잘 알려져 있지 못하여왔다. 그러므로, 이 연구는 옵션가격결정이론의 개념을 이용하여 BOT 프로젝트 파이낸스에서 운영수익상한조항의 가치평가를 위한 이론적 수치모형을 제안하고 이 조항이 사업가치에 미치는 정량적인 영향을 파악하는 것을 목적으로 한다. 이 연구에서는 가상의 BOT 민자사업 도로를 이용하여 제안된 수치모형의 적용성을 확인하였으며 이를 통하여 몇 가지 의미 있는 결론들을 도출하였다. 끝으로, 제안된 수치모형에 의한 결과들을 NPV법에 의한 결과들과 비교·분석하였으며 운영수익상한조항이 사업의 가치에 중대한 영향을 미치는 것을 확인할 수 있었다.