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Pricing the Options in Housing Contracts and Risk Management*

주택거래 계약에 내재된 옵션가치 산정과 위험관리

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Abstract

The recent global financial crisis has hampered economic activities in many ways; however, the impact on the real estate market has been particularly huge. In Korea, falling real estate prices has meant many construction companies going bankrupt because of the vast number of unsold apartments. To alleviate this problem, construction companies have begun to provide potential apartment buyers with free options such as the withdrawal guarantee option, principal guarantee option, premium guarantee option, and buy-back option. In this study, we attempt to analyze the real value and the market risk of these options which has no market value yet. To price the options, we used GARCH option pricing model and Esscher transform. As the result, the price of the withdrawal guarantee option was significantly higher than the other options, and this fact implies that this option may be attractive to the investor in terms of marketing, while the construction firms as issuer may pay the costs. In addition, we applied the DD(Default to Distance) of two companies and VaR of the options in order to manage the risk caused by the options. As the result, according to the analysis of the DD, the issue of the options seems to be helpful to the company immediately. According to the VaR analysis, the principal guarantee option is the most risky option among the options by falling the value of the asset and the value of the option in depressing markets.

Keywords: Global Financial Crisis, House Price Index Derivatives, Risk Management

Ⅰ. Introduction

The recent global financial crisis, which was triggered by a liquidity crunch in the US mortgage market in 2007, rapidly spread worldwide and it is now considered the most severe recession since the Great Depression. As demonstrated by Brunnermeier (2009), the current crisis has similarities to other systemic banking crises in how it was amplified from turmoil in financial markets to economic recession, including negative effects on borrowers'

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balance sheets, narrowed lending channels, runs on financial institutions, and counterparty credit risk. However, by contrast, it has its own distinctive feature, namely that the turmoil was caused by the market's massive reliance on derivatives such as structured and securitized financial securities (i. e., its dependence on the shadow banking system). With the increase in the popularity of securitized off balance sheet securities, a substantial portion of credit risk was possible to pass down to other financial institutions. This meant that banks did not have strong incentives to take particular care when approving and monitoring loan applications, which resulted in cheap credit and a fall in lending standards. This laid the foundations for the crisis.[1)]

The global financial crisis negatively influenced the real estate market globally in three inter-related sections, namely investment, development, and occupational demand[GLA Economics (2011)].[2)] The financial crisis also caused a considerable adjustment in the property market in South Korea as discussed further in Section 3. Moreover, a number of idiosyncratic warning signs in the property market existed before the crisis. The most destructive of these signs was the skyrocketing construction of newly built housing units, which remained unsold mainly in response to government policies, which aimed to depress housing demand.[3)] Moreover, a number of factors challenged the mid- to long-term outlook of the market, including demographic changes, a high level of debt to disposable income ratio in the household sector, and inadequate anticipated housing supply.

It is apparent that the stock piling of unsold housing units in combination with the challenges mentioned above place a question mark on the prospects of the housing market in South Korea. In particular, the former issue has more foreseeable and significant impacts on construction companies because it can ultimately result in a series of bankruptcies in the industry. Therefore, construction companies must look for strategies to overcome these challenges. One such strategy, which is the main topic of the present study, is issuing financial guarantees to potential homebuyers not only as a marketing strategy but also as a risk management strategy.

At present, a few construction companies in South Korea provide potential apartment buyers with free options such as withdrawal guarantee options, principal guarantee options, premium guarantee options, or buy-back options. On one hand, using free options can be considered an effective way to manage the current slumping market condition. However, on the other hand, an increase in the reliance on these off balance sheet guarantees reminds us of what happened from 2007 to 2009, namely the uncontrollable accumulation of risk. This study discusses the usefulness and riskiness of issuing these options.

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1) Refer to Mishkin (2011) for more discussion.

2) The financial crisis has caused not only the breakdown of the credit transmission mechanism but also produced the changes in the value of the financial instruments, especially in the value of the mortgage derivatives. Chen, Chang, Lin, and Shyu (2010) showed that housing price jump risk caused by this subprime crisis changes the valuation of mortgage insurance contracts by assuming that the housing price process follows a jump diffusion process.

3) As discussed further in Section 2, an increase in unsold housing units causes construction companies to undergo serious liquidity problems under the prevailing preconstruction sales system. Although stockpiling unsold housing units eased slightly in 2009, there has been a rapid increase in the number of unsold apartment units since 2004 (24,825 units in 2004 vs. 123,297 units in 2009) and most of those unsold units are located in Seoul and its surrounding regions (97,690 units or 79.2 percent as of 2009).

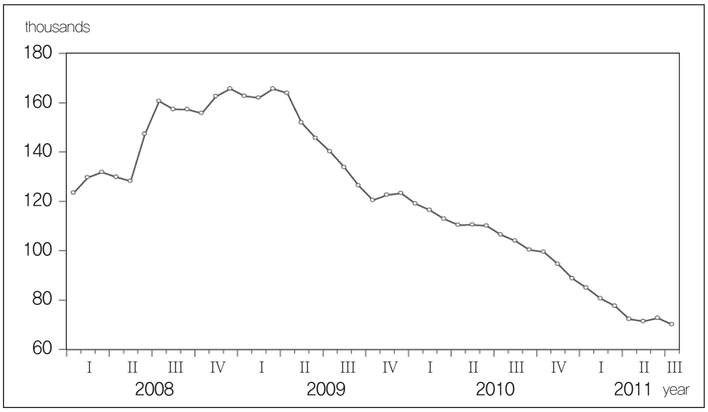
In Section 2, four representative forms of financial guarantees are introduced. The Apartment Purchase Price Index (APPI) is discussed as a proxy of underlying asset price and, using time series analysis, the autoregressive moving average (ARMA) and generalized autoregressive conditional heteroskedastic (GARCH) models are applied on the log difference series of its historical data in order to capture the conditional mean and conditional variance, respectively, in Section 3. The AR(1)–GARCH(1,1) specification is modeled and the conditional Esscher transform is employed to find a risk–neutral measure for pricing these options in Section 4. Finally, the new source of risk is discussed resulting from issuing these financial guarantees free of charge in Section 5.

II. House price index options offered for apartment sales

South Korea has adopted a preconstruction sales system under which project managers recruit occupants at the beginning or at an early stage of their projects for approximately 10 to 20 percent of completion rates.[4] The adoption of this system was supported by the fact that the interests of construction companies, homebuyers, and the government coincided. Under this system, construction companies can finance their projects by collecting installments and securing the demand for housing units under construction beforehand. Homebuyers are also able to gain profits from the difference between sale prices (or listed prices) and market prices, while the government can achieve its housing supply target without providing a high level of financial assistance to the market.

In South Korea, apartments have been termed a low–risk high–yield asset class for the past 30 years or so. However, the market for this asset class has been contracting since the global financial crisis in 2007. Furthermore, strict pricing regulation and increasing debt in the household sector have changed the conception of property as a promising source of investment. Recently, the contraction in the market has been more dramatic among medium– to large–sized apartments, because these are experiencing relatively sharp falls in prices and because they tend to be located in regional areas where housing distribution rates are

Figure 1 _ The number of unsold housing units nationwide(2008～2011)



4) Consequently, project managers are able to finance their projects using the installments paid for by occupants as long as they have obtained both the ownership of sites and a housing guarantee from Korean Housing Guarantee Co. Ltd, a state-owned company. The payable installments consist of the subscription fee, down-payment, intermediate payments, and the remainder. The first three payments have to be below 10 percent, 20 percent including the subscription fee, and 60 percent of the unit price. Therefore, the preconstruction sales system benefits project managers by providing them with a robust source to finance their projects.

already higher than that in metropolitan areas, forcing construction companies to suffer stressful financial difficulties. These financial difficulties are exaggerated by a drop in subscription rates for newly built apartments.

As shown in Figure 1, the number of unsold housing units nationwide peaked at the end of 2008 (165,599 units) and it has been steadily decreasing since the middle of 2009 (70,087 units in the third quarter of 2011).

The new trend and forces mentioned above encourage construction companies to look for various strategies to manage their financial difficulties. One of these strategies is issuing options or guarantees to potential homebuyers in order to promote housing sales. By offering such free options, housing units under construction are considered more attractive to homebuyers since it is possible to transfer the risk caused by the preconstruction sales system to issuers, resulting in an increase in demand. A summary of the free options used by construction companies is presented next.[5]

1. Withdrawal guarantee option

An option holder is allowed to pay a portion of the unit price when signing a contract and to defer the payment of the remainder up to two years. The holder is also entitled to cancel his/her contract without a cancellation charge within six months of the completion of the construction project and 5 to 10 percent of the interest on the installments (or principals) paid accrue depending on the contract period. Moreover, the withdrawal is covered by guarantee insurance and the installments paid as well as the interest accrued are refunded by the issuer upon cancellation. Therefore, this option is an effective vehicle to increase subscription rates, especially when the prospects of the market are so gloomy that the chances of issuing other types of options are limited. Although the issuer of this option can ease its financial difficulties by attracting more homebuyers, the risk transferred to it can be uncontrollable if a large number of contracts are cancelled unexpectedly. Moreover, this option can be abused by speculative investors during periods of low interest rates. The cancellation of contracts also causes losses in terms of opportunity cost.

2. Principal guarantee option

A holder of this option is entitled to receive a predefined amount of money from the construction company or

5) Unfortunately, the volume of the options issued cannot be clearly identified for the following reasons. First, these options are issued as a marketing strategy by a few construction companies; thus, options are not tradable in the market. Second, these options are unregulated and firms have no obligation to report them; therefore, it is hard to obtain exact statistics on the outstanding quantity of them. Furthermore, firms that issue options are mostly small or medium sized, and the financial soundness of these companies is usually weak; therefore, they do not want to report the option volumes for fear of the resulting effect on their reputations in the market. Nonetheless, we tried to collect and analyze the statistics of the options using a key word search including "options, premium, buy-back, principal, unsold and so on" from the major newspapers. We observed that the types of options issued have evolved according to financial and market status, especially during the four distinctive periods of large financial events in the Korean housing market: the Asian financial crisis in 1997, credit card crisis and new governmental regime in 2004, and global financial crises in 2007/2008 and in 2011. The principal guarantee option, which guarantees "no loss for holding apartments" dominated (it was the only type issued in1997 and 2004). However, this option was not enough for clients during the global financial crisis in 2007/2008. Homebuyers asked construction companies to guarantee their "potential profit" in this distressing time. Therefore, premium guarantee options became popular.

cancel the housing purchase contract when the market price falls below the listed price. Upon cancellation, only the principals paid are returned. Like the withdrawal guarantee option, this option has the disadvantage that the issuer becomes unable to manage claims to refund installments when a large number of contracts are cancelled simultaneously.

3. Premium guarantee option

An issuer of this option guarantees to pay a predefined premium above the initial sale price to the holder or to give discounts while the holder pays the remainder if the unit is not traded at a predetermined level of premium within six to 12 months of the completion of the construction project.[6)] This approach secures returns on investment to the option holder and, therefore, improves confidence among investors.

4. Buy-back option[7)]

A holder of this option has the right to resell the apartment to the issuer after having lived there for two or three years. Although this option has its own advantages compared with other types of options, it is usually issued by a low credit-rated or financially vulnerable company that is experiencing liquidity problems. This means that this option tends to be issued by a company facing a high level of financing risk and/or default risk in the situation where an inflow of funds is needed urgently within a short period. Indeed, a number of cases have even been reported where some builders in financial difficulties saved themselves from bankruptcy by issuing this option.

III. Data

There are a number of modeling issues when using apartment sales data in order to use apartments as the underlying assets. First, transaction data are inadequate for asset pricing modeling compared with stocks or bonds trading data because they show illiquidity in transaction frequency and large amounts of trading costs. Moreover, the fact that apartments attract investors as well as homebuyers should be taken into account. In particular, in South Korea small apartments are more often considered residential housing, while speculation plays a significant

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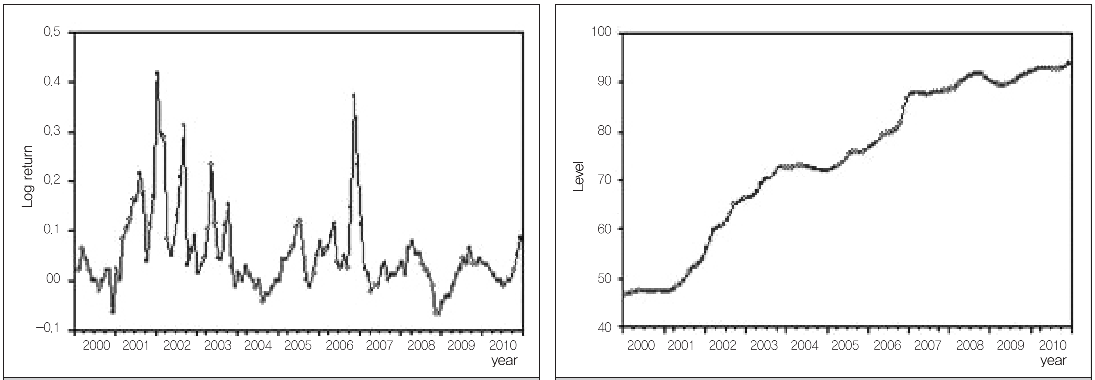
6) The premium above the initial sale price is usually determined as between Korean won (KRW) 30,000,000 and KRW 50,000,000 above the listed price. For example, if the predetermined premium amount was KRW 30,000,000, the issuer would have to pay up to that amount if the market price falls or does not rise to that amount.

7) Ong, Ooi, and Sing (2000) and Sing, Ong, and Sirmans (2003) introduced asset-backed securitization (ABS) deals and a buy-back option embedded in the ABS in Singapore, which is a kind of call option giving the potential of the real estate to the bondholder. Sing, Ong, and Sirmans (2003) utilized a binomial option pricing framework and calculated the explicit prices of the embedded buy-back option.

Figure 2 _ Relative return and the APPI(2000~2010)



Log return of APPI | Level of APPI

role in the market about their large-sized counterparts. In other words, the size of apartments should be carefully examined, and this is partly supported by data that show that the prices of large apartments are more volatile. Next, if an index is used as a proxy, it is important to check that the proxy tracks the movement of actual sale prices accurately. Finally, because the APPI data is low frequency and illiquid, it may be difficult to catch up the change of financial environment immediately.

Incorporating the above issues, time series data of the APPI maintained by Kook Min Bank are selected as the underlying assets.[8] This index is based on nationwide apartment sale prices since 1986. Figure 2 shows the relative return (or log return) and the development of the price index from 2000 to 2010.

It is possible to analyze the business cycle in the housing market using a Markov regime-switching model and its development over time with the log difference series of the index. As a result, a number of characteristic features in the market can be found for the past 10 years. As shown in the left panel of Figure A1(Business Cycle in the APPI), there was a recession in 2000 and 2001 because of the bursting of the dot.com bubble followed by a mild downturn in 2004 and 2005 and a negative impact of the recent global financial crisis in 2008 and 2009. The absolute level of the index, however, kept increasing, though several slumps, demonstrating that there was downside rigidity in the market, which is distinct from other assets. It is important to note that the index series indicates downside rigidity in the market because the absolute level of the index keeps increasing regardless of the percentage change.

In the right panel of Figure A1(Percentage Change in the APPI), the series of percentage change shows that this index was a leading indicator of the economy; the market was sturdy by being lower-bounded by −1.64 percent and the impact of the global financial crisis was limited since the percentage change recovers in 2008. Interestingly, the market experienced a severe downturn in 2004 and 2005 in terms of the percentage change of

8) For more information, see http://land.kbstar.com/quics?asfilecode=5023&_nextPage=page=B001478.popular.

the index, which was caused by the credit card debacle in 2003.

As discussed in Chen, Cox, and Wang (2010), in contrast to the efficient market hypothesis, empirical studies of the dynamics of housing prices have found evidence that autocorrelation and mean reversion in housing prices are significant. Therefore, it is necessary to model the APPI using time series analysis, while the ARMA and GARCH structures are implemented in order to capture the conditional mean and conditional variance, respectively. Specifically, the ARMA(R,M) and the GARCH(P,Q) structures are modeled as follows[9]:

$$ARMA(R, M) : X_t + c + \sum_{i=1}^{R} \phi_i X_{t-i} + \sum_{j=1}^{M} \theta_j \epsilon_{t-j} + \epsilon_t \quad \langle 1 \rangle$$

$$GARCH(P, Q) : \sigma_t^2 = d + \sum_{i=1}^{P} a_i \sigma_{t-i}^2 + \sum_{j=1}^{Q} \beta_j \epsilon_{t-j}^2 \quad \langle 2 \rangle$$

where $\{\epsilon_t\}$ is an independent and identically distributed (iid) normal distribution with mean 0 and variance 1, and R, M, P, and Q represent the order of autocorrelation, moving average, GARCH, and ARCH effects, respectively.

The purchase price index turns out to be a difference stationary series since, using the augmented Dickey–Fuller (ADF) test in order to incorporate its trend into the intercept, it fails to reject the null hypothesis with the index series itself (t-statistics: 0.226862; p-value: 0.9739), while suggesting to reject the null with its first-difference series (t-statistics: -6.785402; p-value: 0.0000). By performing the Box–Jenkins procedure and examining the resulting correlogram, the ARMA(1,0) structure (or the AR(1) structure for simplicity) is selected and additional statistical tests are carried out with the residuals in order to check whether any ARCH/GARCH effects remain.

The autocorrelation structure in Figure A2 shows that the log difference series of the APPI follow the AR(1) structure in the conditional mean.

Table 1 summarizes the outputs of the AR(1)-GARCH(1,1) specification for the log difference series from January 1989 to June 2011 (270 monthly observations) under the assumption of Gaussian innovations.

For the volatility structure

Table 1 _ Estimated coefficients and test statistics of AR(1)–GARCH(1,1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
c	0.001805	0.001148	1.571977	0.1160
ϕ	0.7892***	0.040384	19.54222	0.0000
Variance Equation	Coefficient	Std. Error	z-Statistic	Prob.
d	2.57E-06***	6.15E-07	4.185775	0.0000
β	0.4662***	0.094940	4.910871	0.0000
α	0.5309***	0.072173	7.355988	0.0000

Note: Persistency parameter, $\lambda(=\alpha+\beta)$ shows a strong dependency between the past and future volatility. This phenomenon is well documented in the finance literature and the persistency indicates that the apartment index employed in this study behaves as similar to the financial market equity index.

9) It is assumed that the covariance stationary condition is met. For details, see Chen, Cox, and Wang (2010).

of the series, the GARCH(1,1) structure is chosen through the ARCH LM test and correlogram analysis. It is important to note that the volatility of this series has a tendency to persist because the sum of the two parameters (α and β) determine whether the persistency of volatility is close to one, which is a typical feature of high-frequency financial data. Based on the analysis shown in Figure A3, the AR(1)-GARCH(1,1) model structure is shown to be the most suitable for modeling the APPI.

Ⅳ. Pricing the apartment price index options

GARCH option pricing models have been popular and useful for modeling volatility clustering in asset prices. After Engle (1982) and Bollerslev's (1986) initial contributions, Engle and Mustafa (1992) studied the implied conditional volatilities from the prices of options and Duan (1995) developed a discrete-time option pricing model and its delta in the context of GARCH. However, since most have no closed-form solutions except Heston (1993) and Heston and Nandi (2000), several computational methods such as Duan and Simonato's (1998) empirical Martingale simulations and Ritchken and Trevor's (1999) lattice algorithm have been devised. In addition to the computational problems, an arbitrage-free proof of the GARCH option pricing model complying with that of Black and Scholes (1973) and Merton (1973) can be found in Kallsen and Taqqu (1998). This is accomplished using the geometric Brownian motion to connect the discrete-time GARCH model. Giovanni, Engle, and Mancini (2006) refined their GARCH options in incomplete markets. Cao and Wei (2010) suggested valuing housing index derivatives based on an equilibrium valuation framework in order to avoid the non-tradability of housing indices. Our methodology keeps the risk-neutral framework by eliminating the market price of risk from the underlying equation.

The Esscher transform is a time-honored instrument in actuarial science that was introduced by Esscher (1932). Recently, a number of studies of actuarial science have applied it to pricing financial securities in an incomplete market. Gerber and Shiu (1994) and Siu, Tong, and Yang (2004) applied the Esscher transform in order to value derivatives securities under primitive securities governed by various stochastic processes including the Weiner process. Chen, Cox, and Wang (2010) employed the Esscher transform in order to develop a pricing framework that could investigate the sustainability of the Home Equity Conversion Mortgage program in the US.

Let S_t be the time series data of the monthly APPI from January 2000 to December 2010 and Y_t be the log return $\ln(S_t)-\ln(S_{t-1})$of the data S_t. Then, the first-difference series of the log return $DY_t=Y_t-Y_{t-1}$ becomes stationary and the ADF test (-1.413021 in the level, -5.324349 in the first-difference form) supports this result.

Let $(\Omega, \Im, P)$be a complete probability space, where P is a statistical probability measure and $\Im$ is the filtration related to the development of the APPI. Let T be the expiration of the option. Bühlmann, Delbaen, Embrechts, and Shiryaev (1996) introduced the conditional Esscher transform, which is generalized to stochastic processes. In this study, the conditional Esscher transform is defined as:

$$f^{*}_{X_t}(x\,|\,\Im_{t-1})=f_{X_t}(x\,|\,\Im_{t-1})\frac{\exp(\lambda_t x)}{E[\exp(\lambda_t x)\,|\,\Im_{t-1}]} \qquad \langle 3 \rangle$$

where f_X and f^{*}_X represent the probability density function of before and after the exponential operation and $\Im_{t-1}$ represents the information set containing all the information up to time $t-1$. Then, it can be proved that, for some constant λ_t and

$$A_t=\prod_{k=1}^{T}\frac{\exp(\lambda_t Y_k)}{E[\exp(\lambda_t Y_k\,|\,\Im_{t-1})]} \qquad \langle 4 \rangle$$

$\{A_t\}_{t=1,2,\ldots,T}$ is a Martingale. Let P_t be the restriction of the measure $P:=P_t$ on the information set $\Im_t$. In order to construct a risk-neutral probability measure Q, which is equivalent to the physical measure P on $(\Omega,\Im)$, we choose $\lambda_t=\frac{r-\hat{\mu}_k}{\sigma_t}-\frac{1}{2}$, $t=1, 2, \ldots, T$, and take $dQ_t=A_t dP_t$, $Q_t=Q_{t+1}\,|\,\Im_t$, and a probability measure $Q=Q_T$ on $(\Omega,\Im)$. Then, it is shown that:

$$E^{Q_t}[\exp(Y_t)\,|\,\Im_t]=\exp(r) \qquad \langle 5 \rangle$$

where r is the risk-free interest rate. Consequently, under the risk-neutral measure Q, it can be shown that $Y_t\,|\,\Im_{t-1}\sim N(r-\frac{1}{2}\sigma_t^2, \sigma_t^2)$.

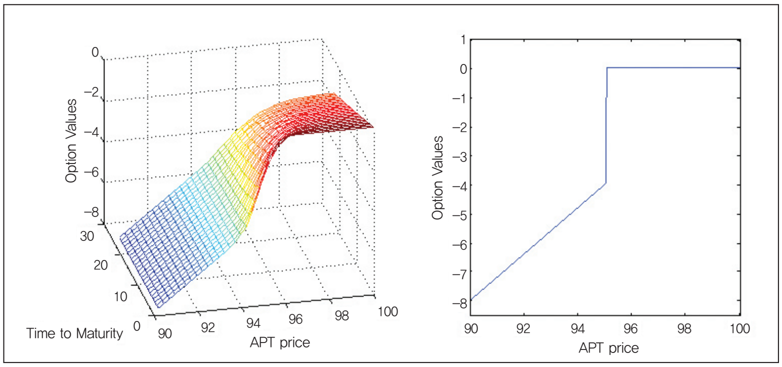
In Section 3, we showed that DY_t fits to the AR(1)-GARCH(1,1) structure; i.e., $DY_t=\phi DY_{t-1}+\epsilon_t$, $\epsilon_t\,|\,\Im_{t-1}\sim N(0, \sigma_t^2)$ and $\sigma_t^2=w+\alpha\sigma_{t-1}^2+\beta\epsilon_{t-1}^2)$. Although this model specification is general, we choose the Heston and Nandi (2000) model as our basic GARCH(1,1) model because some option prices can be obtained in closed form. The volatility process in the model of Heston and Nandi (2000) is suggested as $\sigma_t^2=w+\alpha\sigma_{t-1}^2+\beta(z_t-r\sigma_t)^2$ where $z_t\sim N(0,1)$ in the physical measure and the risk-neutral version of this volatility structure can be written as $\sigma_t^2=w+\alpha\sigma_{t-1}^2+\beta(z_t^*-r^*\sigma_t)^2$.[10] The estimated parameters of the given model are shown in Table 2. With this underlying process specified, the option formulas for the financial guarantees discussed in this study can be expressed

Table 2 _ The estimated parameters of the AR(1)-GARCH(1,1) model

Phi(ϕ)	Alpha(α)	Beta(β)	Gamma(r)	Omega(w)	Current GARCH Variance σ_{t+1}^2
0.7095	0.9010	2.3611E-06	5.33	1.5831E-06	2.897E-05

10) The risk-neutral process has the same form as does the real process with λ replaced by -0.5 and replaced by $r^*=r+\lambda+0.5$ in Proposition 1 of Heston and Nandi (2000).

Figure 3 _ Payoff graph of a short position of the withdrawal guarantee option



as a combination of the three traditional options: the American digital put option, the European put option, and the European digital put option.

1. Withdrawal guarantee option

A holder of this option is entitled to cancel his/her contract at any time before moving in (T_1) without incurring a cancellation charge. Upon cancellation, the installments (K_t) paid as well as the interest (int_t) accrued from time 0 to t are refunded by the issuer. The amount of installments is bound by the terms of the contract and it is generally set to be a proportion of the initial sale price (K_0), namely $a \cdot K_0$, where $0<a<1$. An individual homebuyer is considered to lose his/her interest to hold this option and cancel the contract when the actual sale price at time t (S_t) is below a trigger value (E), which is the minimum acceptable price. Although each individual has his/her own lower bound, meaning that the trigger value is likely to be a random variable, it is assumed to be constant in this study for simplicity.

Therefore, the issuer can be considered to have a short position of an American-style put option, maturing at time T_0 $aK_0 \cdot int_t$ with a trigger value of E through stopping time $\tau=\inf\{0<t<T_1 : S_t<E\}$. Specifically, the payoff of this option is a combination of an American digital option paying for interest and an ordinary American put option paying $(a(K_0-S_\tau))^+$ with as the strike price where $(x)^+$: $\max\{x, 0\}$. The payoff of this option is graphically represented in Figure 3.

At time t, the value of an American put option expiring at time T_1 with a strike price E where $t<\tau<T_1$ is formulated by:

$$P^{AD}(t, E)+E^Q[e^{-r(r-t)}(K_t \cdot int_t+(a(K_0-S_\tau))^+ \cdot 1_{E-S(\tau)>0}], \text{ where } 1_{E-S(\tau)>0}=\begin{cases}1, & \text{if } E-(S_\tau)>0\\ 0, & otherwise\end{cases} \quad \langle 6 \rangle$$

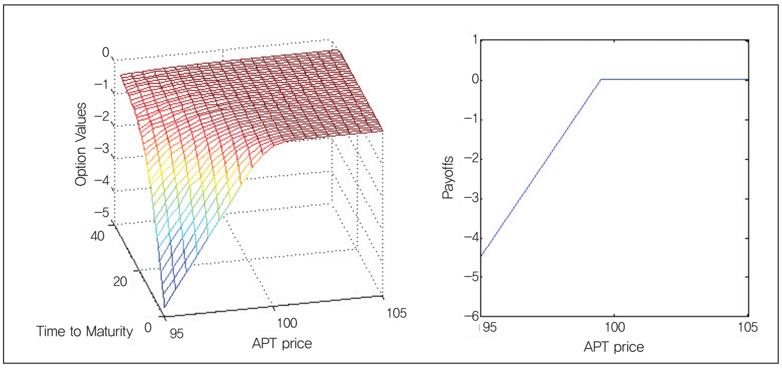
2. Principal guarantee option

Assume that an apartment is sold at time 0 for. K_0 With the principal guarantee option, if its sale price at time $T_1(S_{T_1})$ falls below, the difference between the two prices ($K_0-S_{T_1}$) is returned to the option holder. Therefore, the issuer can be considered to have a short position of a European put option, maturing at T_1 with the exercise price K_0. The payoff is illustrated in Figure 4.

At time t, the value of a European put option with strike K_0 that expires at time T_1 is formulated by:

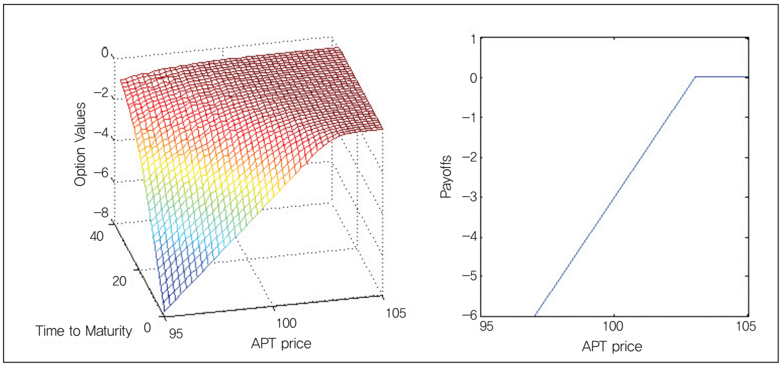
$$P^E(T_1-t, K_0)=e^{-r(T_1-t)} E^Q[(K_0-S_{T_1})+] \quad \langle 7 \rangle$$

Figure 4 _ Payoff graph of a short position of the principal guarantee option



3. Premium guarantee option

Figure 5 _ Payoff graph of a short position of the premium guarantee option (guaranteeing the entire loss)



An issuer of this option guarantees a premium of when the option holder moves in while anticipating the sale price will increase, namely the issuer expects the sale price when moving in (S_{T_1}) will be higher compared with the initial sale price at contract (K_0). As a result, this option can be categorized by whether the entire loss below the premium $(K_0+a-S_{T_1})_+$ or the loss up to the premium is $(K_0+a-S_{T_1})_+-(K_0-S_{T_1})_+$ guaranteed.

1) Guaranteeing the entire loss

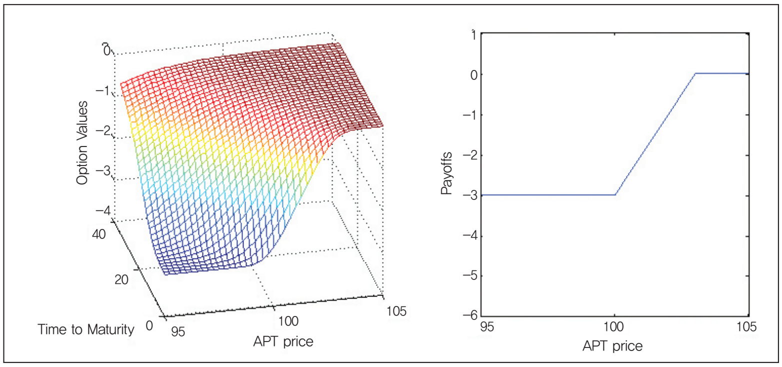
If the option guarantees the entire loss when S_{T_1} falls below (K_0+a), the issuer can be regarded as having a short position of a European put option maturing at T_1 with an exercise price of (K_0+a), which pays $(K_0+a-S_{T_1})$ at maturity. The payoff at maturity is shown in Figure 5.

The value of the premium guarantee option should be $P^E(T_1-t, K_0+a)$ using the notation in ⟨7⟩.

2) Guaranteeing the loss up to the premium

If the option guarantees the loss up to the premium when S_{T_1} falls below (K_0+a), the issuer can be regarded as having a short position of a European put spread option maturing at T_1 with exercise prices of (K_0+a) and K_0, which pay $(K_0+a-S_{T_1})_+-(K_0-S_{T_1})_+$ at maturity. The payoff of a short position at maturity is illustrated in Figure 6.

Figure 6 _ Payoff graph of a short position of the premium guarantee option (guaranteeing the entire loss)



This spread option is the combination of two European put options that have different strike prices, where the value of the premium guarantee option equals $P^E(T_1-t, K_0)-P^E(T_1-t, K_0+a)$.

4. Buy–back option

After moving in at time T_1, a holder of this option has the right to resell the apartment unit to the issuer if the sale price at time T_2 falls below the initial sale price (K_0). Therefore, it can be considered that the issuer has a short position of a European put option maturing at T_2 with an exercise price K_0, which pays $(K_0-S_{T_2})_+$ to the option holder. The payoff of this option is the same as that of the principal guarantee option except for the timing of the contract and maturity; thus, its payoff graphs and more analysis except pricing the option in Table 3 are omitted. The value of the buy–back option is expressed as follows:

$$P^E(T_2-T_1, K_0)=e^{-r(T_2-T_1)}E^Q[(K_0-S_{T_2})_+] \quad \langle 8 \rangle$$

The price of each option is estimated under the assumption that the underlying asset price follows an AR(1)–

Table 3 _ Parameters and options prices

Parameters \ Options	Withdrawal guarantee	Principal guarantee	Premium Guarantee* Type a	Premium Guarantee* Type b	Buy-Back Option
Spot Price (S)	100	100	100	100	100
Strike Price (K)	100	100	100	100	100
Risk Free Rate (month)	3.45%/12	3.45%/12	3.45%/12	3.45%/12	3.45%/12
Time to Maturity (month)	12	36	36	36	36
Deposit ratio	80%	N/A	N/A	N/A	N/A
Interest rate of Depos.	5%/12	N/A	N/A	N/A	N/A
Deposit(=K*.8)	80	N/A	N/A	N/A	N/A
Trigger value	95	N/A	N/A	N/A	N/A
Premium	N/A	N/A	3	3	N/A
Option Prices	3.5525	0.0046	0.0364	0.0319	0.0054

Note: * Type a and Type b of the premium guarantee option denote the option guaranteeing the entire loss and the option guaranteeing the loss up to the premium amount, respectively.

GARCH(1,1) process. In pricing, there are two groups of parameters. The first group of parameters is applicable to all options, and these are listed in all columns. The other is specific to a certain option and only relevant parameter values are recorded. The parameters used in this study are summarized in Table 3.

V. Risk management

1. Risk sensitivity analysis

This section discusses the rationale behind the growing popularity of housing index options using the classical Greeks analysis by focusing on the aspect of information sensitivity.

As discussed in Section 3, the APPI, which is the proxy of the underlying assets, has continued to increase since 2000 without showing a severe downturn. As a result, the development of the index influenced the risk management patterns of the issuers and buyers of options. Generally, if the underlying asset price shows downward rigidity for an extended period of time, the issuer becomes insensitive to the movement of its price and this pattern can be explained using the Greeks analysis. Moreover, the Greeks are vital tools in risk management because they provide the issuer with information about the sensitivity of the option price that results from movementsin the underlying asset price, time value of money, interest rate, and implied volatility. The information sensitivity discussed in this section is realized through two paths: time to maturity and the movement in the underlying asset price.

First, there is a path in which the sensitivity of the Greeks becomes more dramatic as time approaches maturity. It is also possible to notice in Figures 7, 8, and 9 that the three measures of sensitivity – delta, gamma, and theta – become more dramatic as time to maturity decreases, resulting in the option price becoming more sensitive correspondingly. Second, both the issuer and the holder have different motivations to the changes in sensitivity measures according to the change in the level of the underlying asset price, and this study provides a rationale for this path in terms of information sensitivity. The Greeks of the withdrawal guarantee option are illustrated in Figure 7.

The left figure shows how delta responds to a change in maturity and apartment (APT) price. Since construction companies are in the short position, positive delta, negative gamma, and positive theta are shown in the figures. These characteristics are the same in the other figures below. Owing to the properties of the APT price index, typical delta hedging is not considered and thereby risk analysis related to delta is impractical.[11] However, it is worth watching carefully the delta dynamics in terms of information analysis. The center figure of Figure 7 shows the change in delta, namely the gamma dynamics, and the right figure shows the theta that represents

11) Although the risks of holding the short positions are conventionally managed through dynamic hedging using replicating portfolios, construction companies cannot follow this strategy since the underlying assets have not yet been constructed, their transactions are illiquid (although they have been completed), and the index used as a proxy is not traded in the market.

Figure 7 _ The Greeks of the withdrawal guarantee option

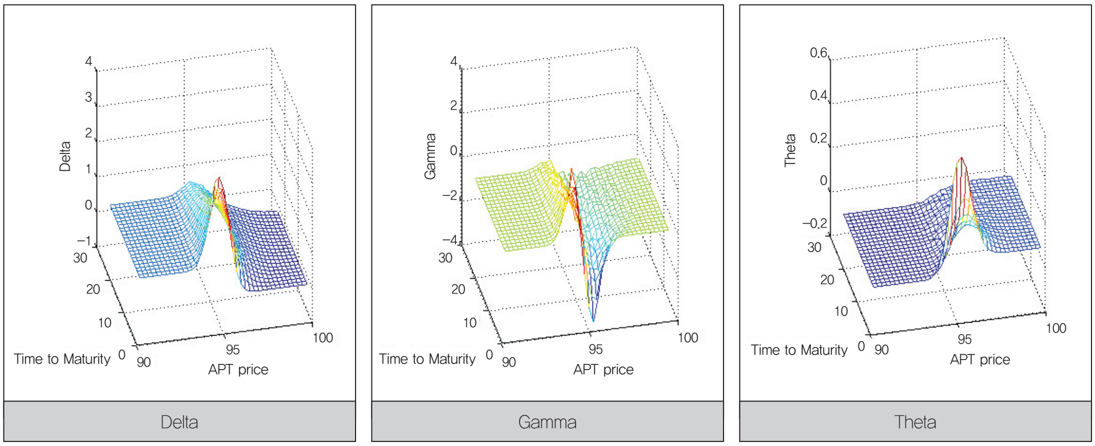


Figure 8 _ The Greeks of the principal guarantee option

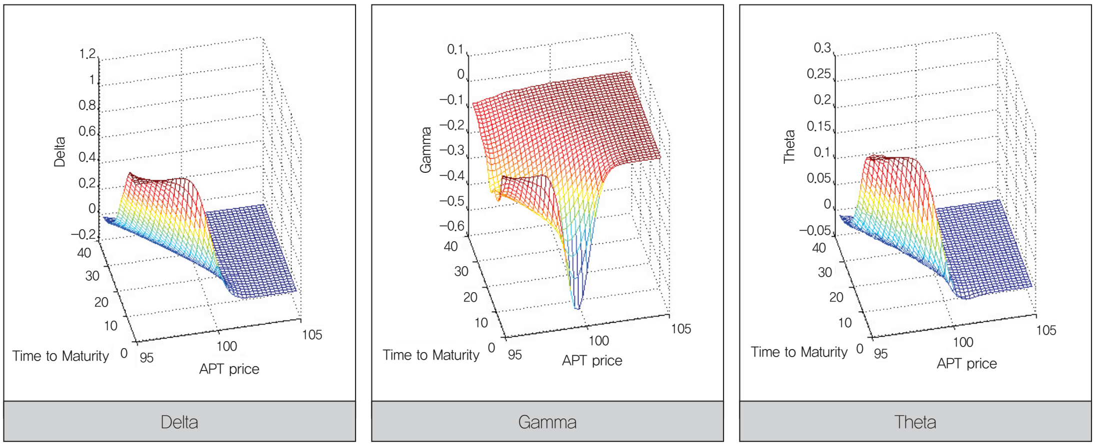
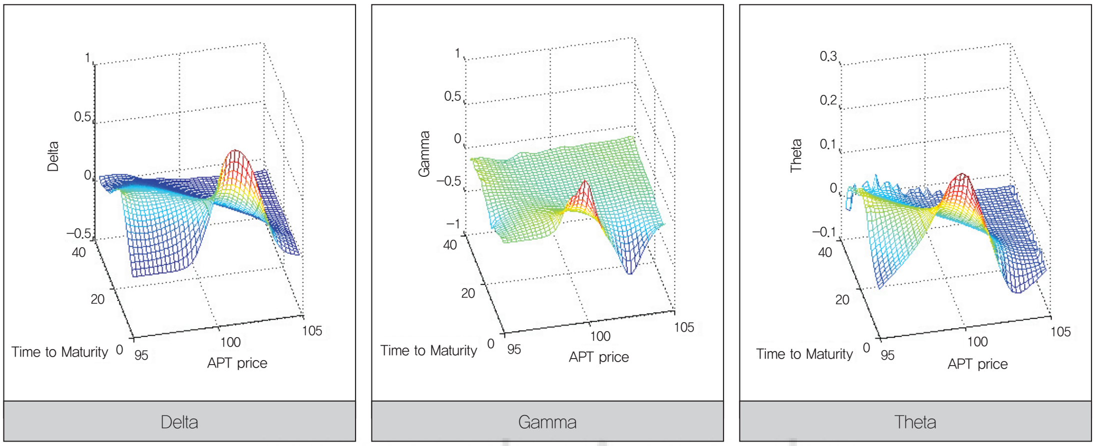


Figure 9 _ The Greeks of the premium guarantee option



time decay, which is the hypothetical gain of the option in the short position. From the analysis of the Greeks of the withdrawal guarantee option, both out of the money (OTM) and into the money (ITM) show no dramatic movement, indicating that risk management is required in the at the money (ATM) area.[12)]

Unlike the analysis of the Greeks of the withdrawal guarantee option, the Greeks of the principal guarantee option show a somewhat biased movement in both delta and theta, indicating that risk management should be focused on the ITM area (Figure 8). Except this point, most characteristics are similar to those of the Greeks of the withdrawal guarantee option. The Greeks of the premium guarantee option show similarities with the analysis of the Greeks of the withdrawal guarantee option. However, risk management should be also considered both in the OTM and in the ITM areas (Figure 9).

Even though it is not possible to construct a delta-neutral portfolio because of the characteristics of the house price index, the three Greeks – delta, gamma, and theta – are important risk management measures for construction companies. In particular, the delta of a currently traded option is sensitive to the movement of the index because the option's price tends to jump when the underlying asset price moves from the OTM level to the ATM level.

Because the index shows a downward rigidity, construction companies may have previously been less motivated to manage risks. However, the declining prospects of the market mean that construction companies have to manage the risks caused by fluctuations in the index or a long-term slump in the market. Moreover, they have to take two more measures (i.e., gamma and theta), which represent the sensitivity of delta and time decay, respectively. These two measures should be analyzed at the same time since they have different signs.

When the underlying asset price remains high during the life of a put option, theta is the only measure that the issuer of the option must take into account and the short position gains profit by the amount of the time value of money as time passes. However, this profit is not actually realized since the options are issued for free as a marketing strategy. When the underlying asset price is at the OTM level, both the issuer and the buyer are better off with the free options and neither is motivated to collect additional information, maintaining a win-win situation. However, if the underlying asset price falls from the OTM level to the ATM level, both parties become extremely anxious about obtaining price information, causing the options to become information-sensitive instruments abruptly. Therefore, unlike in the win-win situation, the issuer has to manage the risks caused by the movement in the underlying asset price over the course of the option's life in this new situation.

2. Default risk management: A case study

Construction companies have attempted to provide various types of financial guarantees for homebuyers not

12) When the underlying asset price remains high, the asset price is likely to be in the OTM level for a put option issuer.

only as a marketing strategy but also as a risk management strategy in order to reduce the number of unsold apartment units. Cash inflow into a construction company consists of its own equity capital, loans from financial institutions, and payments from homebuyers. Recently, the apartment sales market has experienced a slump after the global financial crises. This market condition is gradually leading to a series of bankruptcies affecting small and low credit-rated companies as well as large financially healthy companies. This is because stockpiling unsold apartment units under the current frozen market condition means that the third source of the cash inflow becomes limited to a large extent, causing those companies to suffer serious financial challenges. Therefore, construction companies began to provide tools to transfer the risks associated with real estate trade in order to reduce the number of unsold apartment units and thus to increase their cash influxes from homebuyers. In other words, these options started to gain popularity as a vehicle to manage default risk. Therefore, issuing these options is closely related to the default risks of construction firms.

In order to estimate the default probability of a company, the model in Merton (1974) is employed. Merton calculated the probability of the default of a company at time t using the following formula:

Default Probability = N(−DTDt),

Where N(·) denotes the cumulative distribution function of the standard normal random variable and the distance-to-default, DTDt, is defined by

$$\mathrm{DTDt}=\frac{\ln(V_t/\mathrm{L})+(\mu-\sigma^2/2)(T-\mathrm{t})}{\sigma\sqrt{T-\mathrm{t}}} \qquad \langle 9 \rangle$$

where the asset value V_t of a company follows the geometric Brownian motion with drift μ and volatility σ, and L denotes the amount of liability with maturity T.

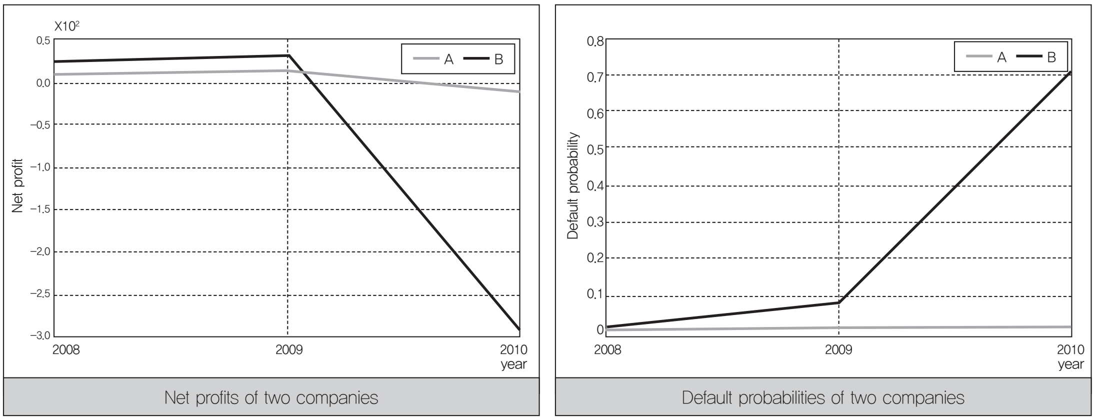
As a case study, the present study examines two construction companies (A and B in Figure 10). These companies initially had similar degrees of financial stability and probabilities of default (i.e., before the global financial crisis in 2007). Company A adopted an active marketing strategy as well as a risk management strategy by providing buy-back options. By contrast, company B remained passive without attempting any novel strategies. As a result, company A was able to stabilize net profit and default probability until 2010, especially when the impact of the financial crisis became devastating in 2009, while company B failed to avoid serious management challenges in terms of net profit and default probability.

However company A has also experienced financial difficulties due to the continuing decline in the price of apartment that is a serious source of risk to the apartment option providers as discussed in the next section.

3. A new source of risks from issuing options

Providing financial guarantees to homebuyers as a marketing strategy incurs a potential source of risk by the

Figure 10 _ Net profits and default probabilities of two companies



Note: The source of the data was closed by the request of the two firms.

issuing company when house prices drop.[13] Therefore, it is important to analyze the level of risk incurred, and this risk analysis is carried out in this subsection using a simulation with the same parameter sets used in Section 4.[14]

First, the prices of the withdrawal guarantee, the principal guarantee, and the premium guarantee (Type b) options are simulated in Table 4, assuming that house prices collapse by 5, 10, or 15 percent after either three or nine months without varying the other parameters. Table 4 shows that when house prices fall by 5 percent, the price of the withdrawal option does not increase. This is not only because this option is not likely to be exercised for being lower bounded to hold it by 95 percent of the initial underlying price, namely being in the ATM condition, but also because the level of contingency vanishes. In addition, the interest accrued on the installments diminishes as time to maturity decreases. However, if the underlying asset price plummets by 10 or 15 percent, the option price directly reflects this plunge because the option is exercised early. This price usually becomes more sensitive to a certain level of depreciation of the underlying asset since the maturity of this option is relatively short (e.g., 12 months). For the case of the principal guarantee option, the impact of a slight fall − 5 percent or less − is limited to the issuer regardless of its timing. However, it becomes riskiest in the relative change in prices, corresponding to a sharp collapse in the underlying asset price, such as 15 percent in this study, because it bears the burden caused by the depreciation of the underlying asset completely. Unlike the principal guarantee option, the loss ensured by the premium guarantee option (Type b) is bounded by the premium amount, meaning that the impact of a collapse in the value of the underlying asset is limited to the issuer. However, it deteriorates compared with

13) Overall, the housing market in Korea is expected to experience a short-term adjustment until 2012. This will be caused by a possible reversal of monetary policy followed by a steady downturn from 2013 to 2017 as baby boomers start to retire. When the number of households aged 35 to 54 begins to decrease nationwide, the housing market is forecasted to plummet regardless of property location or size.

14) We assume that a hypothetical construction company issues the withdrawal guarantee, the principal guarantee, and the premium guarantee options embedded in an apartment sale.

Table 4 _ Risk sensitivity analysis under market collapsing

Type of Option	Initial Price	Rate of Collapse					
		5%		10%		15%	
		Timing to Collapse		Timing to Collapse		Timing to Collapse	
		3 month	9 month	3 month	9 month	3 month	9 month
Withdrawal	3.6768	3.4279	2.0961	8.0000	8.0000	12.0000	12.0000
Principal	0.0044	0.1735	0.3290	1.7913	2.8380	6.0106	7.5545
Premium (Type b)	0.0283	0.5780	1.0006	2.0858	2.5209	2.6987	2.7708

the case of the principal guarantee option through the use of relatively less intense rates of depreciation up to 10 percent.

Next, value at risk (VaR)[15] is simulated in, Tables 5, 6, and 7 at 1 percent and 5 percent probabilities in the issuer's profit and liability distribution, assuming that the drift in the apartment sale price is constant or decreases by 5 and 10 percent per annum. In each case, the losses from both the depreciation in the value of the housing unit and the issuing of an option are incurred by the issuing company. Therefore, this novel marketing strategy places a new source of risk on the issuer in the form of the latter risk.

The withdrawal guarantee option is not likely to be triggered or exercised if the underlying asset is depreciated by 5 percent or less, while the other two options are affected by the fluctuation of the asset throughout their maturities. Specifically, when the apartment sale price is assumed to remain constant, most simulated losses are lower than 5 percent of the asset value, which prevents early exercises of the options issued. As a result, the loss incurred from issuing this option is realized only by the interest accrued on the installments, which leads to the same values of VaR at both probability levels. Moreover, the loss from this source can be higher than the loss from the depreciation of the underlying asset because the former includes the amount of interest to be paid.

As expected, the guarantee that ensures the principal amount can result in huge losses for the issuer. The simulation results for this option indicate that the losses from both these sources affect the issuing company by the same degree regardless of the levels of probability selected, while the loss incurred from issuing the premium guarantee option (Type b) is bounded by its premium amount. It is expected that the loss incurred from the depreciation of the underlying asset will increase if the drift in the apartment sale price declines from 0 percent to −5 percent. Among these options, the loss of the withdrawal option is simulated to be the least because the maturity of this option is only one-third that of the other two options (Table 6). In addition, since this option also guarantees the interest accrued, this proportion is realized in excess of the loss incurred from the asset depreciation.

When the underlying asset is assumed to experience the severest slump (−10 % per annum, Table 7), the losses

15) From the result of Kevin and David (2006), VaR is flawed in several perspectives. We have tried several risk measures and notice that basic arguments in this study do not change, therefore we adopt VaR measure.

Table 5_ Risk sensitivity analysis with VaR (Case of market drift = 0%)

Category	Drift of Apartment Sales Price = 0% per Annum					
	VaR Critical Point					
Type of Option	1%			5%		
	Loss from Depreciation	Loss from Option	Total	Loss from Depreciation	Loss from Option	Total
Withdrawal	5.0869	3.6667	8.7536	3.3918	3.6667	7.0585
Principal	8.5135	8.5135	17.027	6.0411	6.0411	12.0822
Premium (Type b)	8.5135	3	11.5135	6.0411	3	9.0411

Table 6_ Risk sensitivity analysis with VaR (Case of market drift = −5%)

Category	Drift of Apartment Sales Price = −5% per Annum					
	VaR Critical Point					
Type of Option	1%			5%		
	Loss from Depreciation	Loss from Option	Total	Loss from Depreciation	Loss from Option	Total
Withdrawal	6.3674	8.4392	14.8066	5.9141	8.0480	13.9621
Principal	21.2522	21.2522	42.5044	19.0721	19.0721	38.1442
Premium (Type b)	21.2522	3	24.2522	19.0721	3	22.0721

Table 7_ Risk sensitivity analysis with VaR (Case of market drift = −10%)

Category	Drift of Apartment Sales Price = −10% per Annum					
	VaR Critical Point					
Type of Option	1%			5%		
	Loss from Depreciation	Loss from Option	Total	Loss from Depreciation	Loss from Option	Total
Withdrawal	6.7502	8.1858	14.9360	6.3681	7.5798	13.9479
Principal	31.9968	31.9968	63.9936	30.3031	30.3031	60.6062
Premium (Type b)	31.9968	3	34.9968	30.3031	3	33.3031

incurred from the withdrawal guarantee option are lower compared with the case of market drift = −5%.

This is because, under the new drift level, the depreciation of the underlying asset accelerates so that this option is triggered to be exercised earlier in a shorter period time. Therefore, a massive collapse in the apartment sale price causes the early exercises of the options issued so that the amount of losses is prevented from being blown up excessively.

As the final simulation, we considered how counterparty credit risk affects option prices. As shown in Table 8, as the default probability of the issuing company increases, the practical option price decreases as expected. In addition, the effect of the default probability is relatively minor compared with the effect of the drift.

Table 8 _ Effects of the default probabilities of construction companies

Category	Effects of Default Probabilities of Construction Companies			
Default Probability	0.5%		10%	
Drift of Apartment Price	3.45%	-10%	3.45%	-10%
Withdrawal	3.6485	6.3227	3.3002	5.7190
Principal	0.0056	34.8557	0.0050	31.5278
Premium (Type b)	0.0337	4.0293	0.0304	3.6446

Ⅵ. Conclusion

The housing market in South Korea, which has developed under a preconstruction sales system, features downward rigidity and sturdiness despite several slumps. However, the market trend is expected to be reversed by the impact of the global financial crisis as well as a number of domestic risk factors. The use of the current early warning signs for construction companies is alarming because unsold apartment units are stockpiling. This oversupply is intensifying the financial difficulties suffered by construction companies because of a drop in cash inflows from homebuyers. In order to manage this challenge, the concept of providing financial guarantees to homebuyers is gaining popularity as a novel marketing strategy as well as a risk management strategy. Generally, homebuyers can benefit by investing in apartment units and exploiting these options for free, since contingent losses caused by the depreciation of house prices are borne to the issuers. As this marketing strategy attracts more potential buyers, issuing companies are able to reduce their default risks because the ensuing improved cash inflows from customers.

In South Korea, however, the popularity of these options is largely misunderstood because of the historic downward rigidity and sturdiness of the housing market. In fact, issuers are mainly motivated to reduce their high levels of short-term default risks at the expense of more uncertain levels of longer-term price risk based on the speculation that the historic market movement would continue in the future without understanding their risks fully.

In order to analyze the risks associated with issuing these options more extensively, the underlying asset prices were modeled using the AR(1)-GARCH(1,1) model to capture their stochastic properties and the conditional Esscher transform was applied to value the guarantees. According to the result of the analysis, the price of the withdrawal guarantee option is significantly higher than the others. As the reason, it has relatively shorter tenor and higher fixed interest rate than those of the other options under the faltering property market. This option clearly has a strong marketing effect, but it should be financial burden to the company. After pricing the options, their risks were analyzed using the Greeks to assess risk sensitivity, the distance-to-default measure to illustrate the benefit of reducing default risk, and simulation to demonstrate new sources of risk. On the process of measuring DD, we compare the risks of two companies, one did used such options and another did not, and the one used options was able to stabilize their DD significantly than the other. However, we also performed simulation to demonstrate new sources of risk. This emphasizes the possibility that the housing index derivatives issued as risk management and marketing tools can be a new source of future risk.

Although the present research analyzed the risks associated with the financial guarantees extensively, a number of future research topics could complement this study. The issuing patterns of these options should be analyzed in connection with the distributional information of the index and the issuing data on the options. Under the current preconstruction sales system in Korea, the described options are often exercised many years after contracts have been signed. This forward nature of the commitment could entail a credit risk for the issuing party since the construction companies that issue free options are usually small or medium-sized ones. Long-term options that offer interest guarantees are also related to interest rate risks. Furthermore, many homebuyers take out huge loans from banks at variable interest rates. This could lead to financial distress when interest rates soar.

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요약

주제어: 글로벌 금융위기, 주택가격지수 파생상품, 위험관리

최근의 금융위기는 다방면에서 경제적 활동을 제약해왔는데 부동산에서의 충격은 더욱 막대했다. 국내에서 부동산 가격의 하락은 미분양된 아파트 수의 증가로 인한 많은 건설사들의 부도를 초래한다. 이러한 문제를 완화하기 위해, 몇몇 건축회사들은 아파트의 잠재적 구매자들에게 분양취소 보장옵션, 원가보장 옵션, 프리미엄 보장옵션, 환매옵션 같은 무료 옵션을 지급하기 시작했다. 본 논문에서는 가치가 매겨지지 않은 이러한 옵션들이 어떤 시장가치를 가지며, 어떤 위험을 가지는지를 분석하는 데 목적이 있다. 이 옵션들의 가격을 평가하기 위하여 GARCH Option Pricing Model과 Esscher Transform을 사용하였다. 평가 결과, 네 가지 옵션 중 분양취소 보장옵션의 가격이 다른 옵션들보다 월등히 높았으며, 이는 마케팅 측면에서 매력적인 반면, 건설사들에 부담으로 작용할 것으로 보인다. 또한 이 옵션들이 초래할 것으로 보이는 위험을 관리하기 위하여 DD(Default to Distance)와 VaR을 적용하여 분석하였다. 그 결과, DD의 분석에 따르면, 옵션의 발행은 일시적으로 회사에게 도움이 되는 것으로 보인다. 하지만 VaR의 분석에 따르면, 미래의 시장이 침체될수록, 원가보장 옵션이 자산가치 하락과 옵션평가 하락에 의해 가장 위험한 옵션으로 분석되었다.

Figure A1 _ Business Cycle [left] and Percentage Change in the APPI

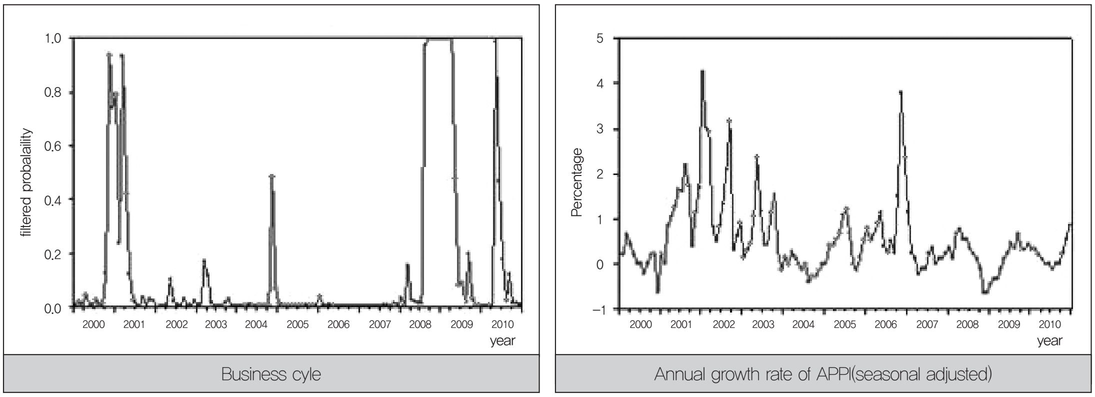

Figure A2 _ Correlogram of the House Price Index

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.735	0.735	166.55	0.000
		2	0.551	0.023	260.46	0.000
		3	0.370	-0.094	302.80	0.000
		4	0.329	00179	336.39	0.000

Figure A3 _ Correlogram of GARCH(1,1) Residuals

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.089	0.089	2.0839	–
		2	−0.016	−0.025	2.1544	0.142
		3	−0.105	−0.103	5.0807	0.079
		4	−0.025	−0.007	5.2448	0.155
		5	0.069	0.070	6.5154	0.164
		6	0.040	0.017	6.9376	0.225