Children's judgments about proportional equivalence with discrete quantities.

I. Introduction

Studies on proportional reasoning ability have covered a variety of tasks, ranging from straightforward judgments about the relative fullness of two different glasses of water (Bruner and Kenny, 1966; Siegler and Vago, 1978) to sophisticated judgments about probability (Piaget and Inhelder, 1975). This ability to judge the relation among relative quantities is an important quantitative skill that may be challenging for young children (Chapman, 1975; Hoemann and Ross, 1971; Karplus, Pulos, & Stage, 1983; Noelting, 1980; Piaget and Inhelder, 1975; Siegler and Vago, 1978). Piaget and Inhelder (1975) claim that this ability does not emerge until early adolescence. This claim comes from the results of their early study, where children had to choose which of two discrete sets composed of red and white marbles has a higher probability of getting a red marble on a random draw. Solving this type of problem includes two factors. First, children have to establish relation between the numbers of red and white marbles for each set (first-order reasoning). Second, children had to think about the "relation between the relations."(second-order reasoning) In the above example, children had to compare the relative numbers of red marbles that each set has. The results of Piaget and Inhelder's study show that children rarely showed any success until the age of 11. Interestingly, the majority of their failure was related to judging on the basis of the absolute number of the target (red) marbles rather than on the basis of the relation between the numbers of the target (red) and non-target (white) marbles. For example, children decided that a set with 3 red marbles and 4 white marbles had a larger probability of getting the red marble than a set with 2 red marbles and 2 white marbles.

This result, contrary to Piaget and Inhelder's original claim, may indicate that children's failure in proportional task was related to the difficulty of using natural numbers to represent proportion rather than to the complete absence of proportional understanding. In fact, most of the previous studies where children failed at proportional task involved countable entities, such as discrete quantities (Piaget and Inhelder, 1975; Noelting, 1980, Jeong, Levine and Huttenlocher, 2003). Young children, who do not know how to use natural numbers to represent proportion, may have failed by inappropriately using natural numbers. This notion predicts that if a task does not provide countable entities, children would not be misled by number information and would be successful in working out proportions through some other type of proportional reasoning strategy.

Indeed, there is accumulating evidence that even young children can successfully judge the relation among proportions when the tasks did not involve discrete quantity and thus did not provide numeric information. (Acredolo, Horobin, Bank, and O'Connor, 1989; Baillargeon, Needham, & DeVos, 1992; Duffy, Huttenlocher & Levine, in press; Goswami, 1989; Huttenlcoher, Duffy, & Levien, 2002; Huttenlocher, Vasilyeva and Newcombe, 1999; Resnick and Singer, 1993; Singer-Freeman and Goswami, 2001;Spinillo and Bryant, 1991). For example, in a map reading study by Huttenlocher, Vasilyeva and Newcombe (1999), even preschoolers can carry out a proportional translation along a single dimension in the context of this simple mapping task. In their experiment, 3-and 4-year old children were given a picture of a rectangle that represented a sandbox. A dot was written in the rectangle to indicate where a toy was hidden in the sandbox. Children were asked to point to the actual location of the hidden object. The results show that all four-year-olds and more than 50 % of three-year-olds successfully translated distance from the scale of the written map to the scale of the actual sandbox. When the task involved continuous quantities instead of discrete quantities, children succeeded by comparing target and non-target amount (part-part reasoning) or comparing target and whole amount (part-whole reasoning) in terms of continuous amount.

The discrete sets allow children to count the number of items so that children count the number. This number (counting) strategy, if used properly, is a highly accurate tool for judgments about proportion, guaranteeing exact responses. In order to represent the numeric relationship correctly, children must count both the number of target items and the total number of items and integrate them in a multiplicative way. This of course can be done by using the conventional system in mathematics such as fractions. However, mathematical instruction on how to use fractions to compare two proportions does not occur until the 5th grade. Rather, young children have strong knowledge about natural number from the tremendous amount of inputs that emphasize it. This strong knowledge about natural number might have misled children to rely on the meaning of separate numbers even for solving proportional reasoning tasks. Thus, the most common mistake that children commit is to respond only on the basis of the absolute number of target items (numerator), rather than on the basis of the relation between the numbers of target and non-target items. However, only a few studies have directly investigated this conclusion.

The current study investigated 4-, 6- , 8- and 10-year-old children's abilities to make judgments about equivalent relations among proportions when the task involves discrete quantities in order to determine whether children's errors were systematic in such a way that the absolute number of target items was misleading children[1]. The matching task involved presenting a target card with a certain number of colored and uncolored discrete items, which was subsequently followed by children's choosing a card with an equivalent proportion from three choice cards. The total number of items in the three choice cards were the same as each other but were different from that in the target card. The number of colored items (focal quantity) in each choice card was manipulated in a way that could reveal how children responded, as follows (see Figure 1 and Figure 2). One of the choices had the same proportion of colored items as the target card, which was the correct response. Another choice card exactly matched the target card only in terms of the number of colored items (absolute number foil). Children's choice of this card would indicate that their failures were related to their matching in terms of the

[1] The reason why we included 4 year olds as our youngest age group was that existing research has shown that they are the youngest children who can successfully reveal their proportional reasoning ability through their verbal responses and the reason why we included 10 year olds as oldest age group was that these are the youngest children who have learned and practiced the appropriate mathematical convention for judging proportions, according to the school curriculum for CPS (Chicago public school). 6 and 8 year olds were included to have a broader picture of this development.

absolute number of colored items without considering the relation between the numbers of colored and uncolored items. The third one matched neither in proportion nor in absolute number to the colored items (random foil). Children's choice of this card may seem to be a random error. However, children's choice of this card would tell us more than this. Selecting this card more frequently than the absolute foil would pinpoint a child's understanding that the absolute quantity (number) does not matter when judging proportions, which is the most important component of understanding proportion. Even though they fail in choosing the right choice due to the lack of ability to make an exact judgment about proportion, their strong knowledge about proportion would lead children to exclude the absolute number foil. Children's selection of this random choice may therefore reflect this kind of understanding.

A second question investigated in the current study is how children's conventional counting skill is related to their ability to make proportional judgments. Particularly, this study examined whether children's proficiency in counting skill is related to the more frequent use of the absolute number strategy by looking at the correlation between their scores in counting tasks and the frequencies of their responses to the absolute number foil. From the age of 3, children can count a

Figure1a.. Proportional matching task (Small to Large direction)

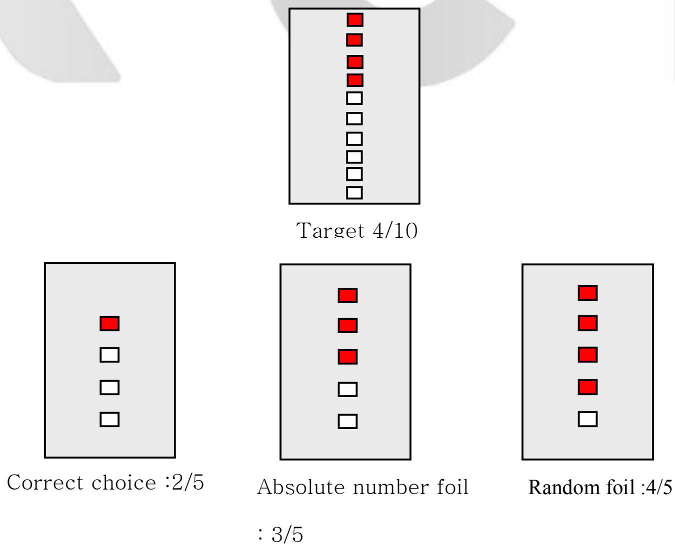

Figure1b. Proportional matching task (Large to Small Direction)

small number of items exactly, using one-to-one correspondence (Mix, 1999; Mix, Levine & Huttenlocher, 1999; Wynn, 1990). Once children learn the effectiveness of number for measuring discrete sets, the counting activity of discrete items become more voluntary and therefore consolidated. Thus, it was hypothesized that children with strong knowledge about counting convention would more likely to count the number of items, resulting in poor performance. For this purpose, 'how-many' and 'give-a number' tasks were executed.

Finally, this experiment also incorporated the role of half-boundary in judging about proportions in order to get a broader picture of early development of proportional understanding. As was shown in several studies, the comprehension of "half" is acquired earlier than other fractions/proportions (Hunting, 1986; Hunting & Davis, 1991). Thus, children's understanding of half can be used for judging understanding other proportions (fractions). For example, before children can represent the exact proportion, say 3/8, they may represent it as "less than half". Spinillo and Bryant's (1991) study actually showed that half boundary plays an important role in making proportional reasoning task. In their study, 4 to 7-year-old children were shown a picture of a rectangle composed of blue and white regions. Children were then presented with two different block models and were asked to choose the one that matched the picture. Most importantly, in half of the problems, children can use a half as a boundary for matching target proportions, because the correct choice and foil are across a half boundary (e.g., 3/8 vs. 5/8), since coding the proportion of blue region as "less than half" without coding the true ratio allow them right choice. In the other condition, children cannot use one-half as a boundary because the two choices are within the half boundary (e.g., 3/8 vs. 1/8). The results show that children in all age groups performed better in the condition where they can use the half-boundary to make their judgments, indicating that the half plays like a boundary for categorical perception for other proportions.

Therefore, the additional question investigated in this study was the role of the half-boundary in judging proportional equivalence with discrete quantities. This was important for several reasons. First, this present study used discrete quantities, whereas Spinillo and Bryant's study used a continuous quantity, such as a block divided into colored and uncolored regions. Thus, this present study could also investigate whether the half-boundary is an important factor when the numerical version of a proportional task is used. Second, manipulating the difficulty of problems using half-boundary can allow us to determine whether or not children consistently use an erroneous counting strategy. Specifically, we can decide whether children would depend on their basic initial ideas about proportion, such as half, instead of erroneously relying on their knowledge about natural numbers when the task was made in a way that half boundary was available. Therefore, the half-boundary was incorporated in constructing test items by including all possible combinations of choices with regard to the use of the half-boundary.

II. Methods

Participants

Eighty children participated in this experiment. They were 20 children in each of four age groups: 4 year olds (mean: 4 years and 6.5 months), 6 year olds (mean: 6 years and 5.5 months), 8 years olds (mean: 8 years and 9 months), and 10 year olds (mean: 10 years 8 months). The children were from 5 preschools and 6 elementary schools in the Greater Chicago area. 4 year olds attended preschools and 6, 8, and 10 year olds were in 1st, 3rd and 5th grade respectively. There were 12 females and 8 males in the 4- year group, 11 females and 9 males in the 6-year group, 9 females and 11 males in 8-year group and 9 males and 11 females in 10-year group.

Materials and Procedure

Each child completed a proportional equivalence matching task and two counting tasks ("How many" and "Give a number" tasks). The matching task was always administrated before the counting tasks and the whole procedure took approximately 25 minutes for each child.

The basic format of the matching task involved the presentation of a target set followed by the child's choice of an equivalent set from three choice cards. Both the target and the three choices were presented on a light gray colored card, measuring 8½ by 11 inches. Across the center of each card, a line of colored and white rectangles was positioned vertically. The sizes of all individual rectangles in the target set were identical (1.6cm by 1.4cm). The three choice cards had the same number of rectangles, but they had a different number of rectangles from the choice card. The size of individual rectangles in the choice cards was different from those on the target cards- the area of the individual rectangle in the choice cards was smaller by 30 % than the area shown in the target card. This design ensured that there was no matching in terms of the absolute amount of colored items.

Most importantly, the numbers of colored and uncolored (white) rectangles in the choice card were manipulated in a way that could show how children would respond. As shown in Figure 1, one of the choices had the same proportion of colored and white rectangles as the target card, which was the correct response. Another card had exactly the same number of colored rectangles as the target card. The third one matched neither in proportion nor in absolute number to the colored items.

As was previously mentioned, the use of a half-boundary was also considered when we construct choices. The problem included all possible combinations of choices with regard to the use of the half-boundary. In a quarter of the problems (CC), both the foils cross the half-boundary. In another quarter of the problems (CW), the absolute foil is in the other half of the correct choice and the random foil is within the same half boundary with the correct choice. In a third quarter of the problems (WC), the absolute foil is within the same half-boundary with the correct choice and the random foil is in the other half of the correct choice. In the fourth quarter of the problems (WW), all three choices are within the same half-boundary. If children truly rely on the half-boundary for representing proportion, the problems in the first category (CC) should be the easiest and the problems in the last category (WW) should be the most difficult. In addition, problems also differed in the direction of set size of target and choice cards: small target set to large choice card or large target set to small choice set. On half of the trials, the total number of rectangles in the target card is smaller than the total number of rectangles in the choice cards (e.g., 2/5 to 4/10: see Figure 1a). On the remainder of the trials, the total number of rectangles in the target is larger than that in the choice card (4/10 to 2/5: see Figure 1b). All problems are presented in Table 1.

Table 1. Problems in the proportional matching task

Direction in Size	Half boundary	Target	Correct	Absolute Foil	Random Foil
Small to Large	CC	3/4	6/8	3/8	1/8
	CW	4/6	6/9	4/9	8/9
	WC	2/5	4/10	2/10	6/10
	WW	2/6	2/12	4/12	5/12
Large to Small	CC	4/12	2/6	4/6	5/6
	CW	4/10	2/5	4/5	1/5
	WC	2/10	1/5	2/5	3/5
	WW	3/12	2/8	3/8	1/8

The equivalence matching task was introduced as a game in which the experimenter told the child, “we are going to make magic water with red and white water." Placing a target card in front of a child, the experimenter then said to the child, "This picture shows how we have to mix red and white water to make the magic water." Presenting three choice cards side by side, the experimenter said, "Now, I want to make the exact same magic water as this, (pointing to the target card) but a different amount. Can you tell me which one of these three can make magic water just like this?" The target card remained in full view of the child while they responded so that there was no memory load in our task. The position of three choice cards was systematically changed, so that the correct choice was located in the center in one-third of the trials. In addition, different colors were used for different trials to prevent children from relying on the representation of previous trials. Test trials were randomly ordered and three different forms of random orders were constructed.

The "How many" and "Give-a-number" tasks was given to measure the child's mastery of the conventional counting system after the equivalence matching task was executed. In the "How many" task, children were given cardboards with four, seven, and ten squares in a horizontal line. Children were asked to count the squares aloud and then were asked to tell how many squares there were. In the "Give-a-number" task, children were given a pile of 15 discs and asked to place a certain number of them on a blank index card. Children were asked to place three, six, and nine circles. Once the child responded, the discs were returned to the pile.

III. Results

Accuracy scores were calculated for each child as a percentage of correct responses on each experimental condition. Average percentages correct for each experimental condition are presented in Tale 2. The inspection of Table 2 indicates that children indeed perform differently according to their ages and half-boundary. A repeated measure analysis of variance on children's performance with age as a between-subject variable and half-boundary as within-group variable were carried out[2]. The results of this analysis revealed significant main effects of both age, $F(3,76) = 21.02$, $p < .001$, and half-boundary, $F(3, 228) = 10.15$, $p < .01$. No interaction was found significant (each $p > .05$).

Table2. Average percentage correct for each experimental

Half	Age			
Boundary	4 year	6 year	8 year	10 year
CC	37.5	35	75*	85*
CW	25	17.5	42.5	70*

[2] All percentage scores were arcsine transformed before conducting analysis of variance.

WC	15	17.5	40	70*
WW	20	12.5	17.5	70*

*: Children's performance differs from the chance (33%), p <.05.

Significant main effects of age revealed that children in different age groups performed differently. Post hoc pair-wise comparisons (Tukey HSD) revealed that 10 year olds (Mean: 73.8 %, SE: 6.63) outperformed children in all other age groups (4 year olds- Mean: 24.4 %, SE: 5.2, 6 year olds – Mean: 20.6 %, SE: 3.5 and 8 year olds- Mean: 41.8 %, SE: 6.6), as was expected. Only ten year olds' performance was significantly above chance ($t(19) = 6.12$, $p < .01$). As is shown in Figure 2, children's performance did not simply improve with an increase in age. The 4 year olds' performance was slightly higher than that of 6 year olds, resulting in a J-shaped function of development. More interestingly, comparing the correct percentage score to the chance level (33%) for each age group, using two-tailed t-tests, revealed that 6 year olds performance differed from chance, but in an opposite direction. Their performance was significantly below chance ($t(19) = -2.59$, $p < .01$). 8 and 4 year olds' performances were not significantly different from the chance level (each $p > .05$).

In order to understand this special pattern of development, the children's actual responses were examined for each age group. There were three response categories for this proportional matching task: the correct choice, the absolute number choice, and the random choice. Figure 3 presents the percentages of

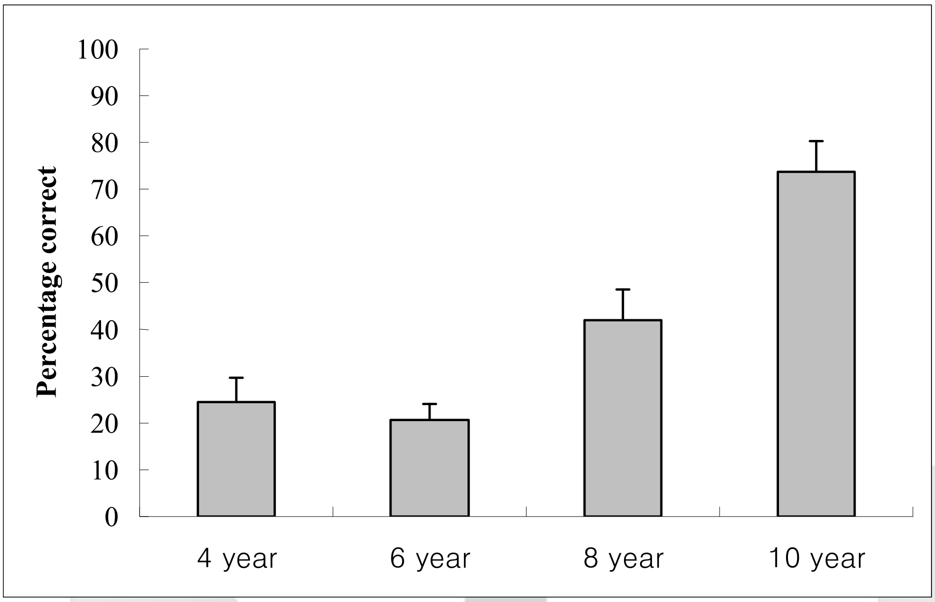

Figure 3. Mean percentages of correct responses on matching task for each age group

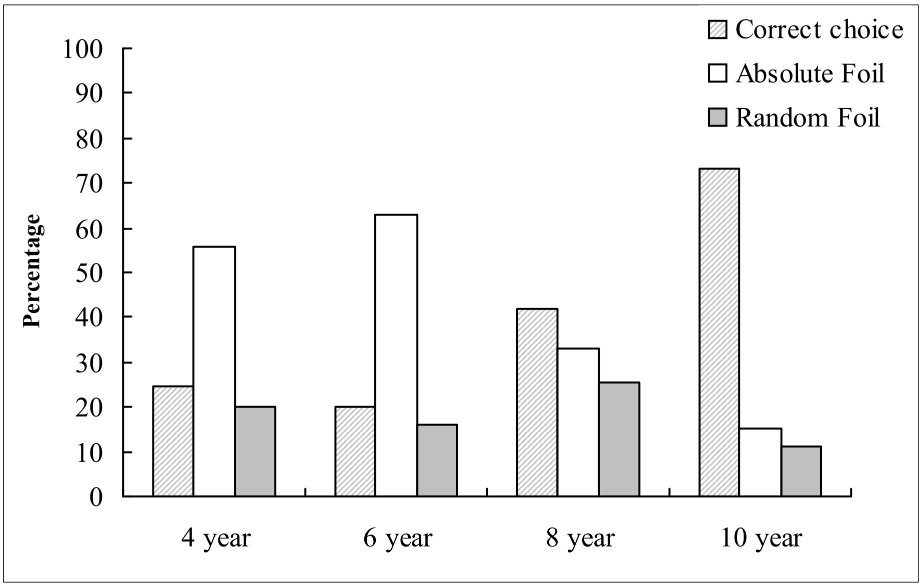

*Figure 4***.** Mean percentage of responses to each choice for different age groups.

children's relative responses among the three choices for different age groups. This graph reveals different natures of children performance for different age groups. 10 year olds chose the correct choice most frequently and 8 year olds did not choose any of the choices most frequently. 4 and 6 year olds, however, overwhelmingly preferred the absolute number choices. The paired t-test revealed that the percentages of responses to absolute foils were significantly above chance (33%) for 4 and 6 year olds (4 year: $t(19) = 5.64$, $p < .01$; 6 year, $t(19) = 4.76$, $p < .01$). The result of 4 and 6 year old indeed confirms the hypothesis that young children's failure is related to their erroneous use of number in proportional reasoning task.

The next question investigated in this study was to explore how young children's knowledge about counting convention was related to their performance on proportional reasoning task. Recall that for testing counting proficiency, two different counting tasks were used. For the " How Many" task, children received one point for counting correctly and received another point for giving the last count word for each card, resulting in a total score of 6. For the "Give-Number" task, children received one point for each correct response, resulting in a total score of 3. Performance on the How many and Give-Number task was significantly correlated ($r = .65$, $p < .01$), providing convergence evidence that a common underlying skill was measured by the two tasks. Thus, all points earned were pooled together to yield a score for the conventional counting skill. The 8 and 10 year olds received the full score on the counting tasks. There was some variation in 4 and 6 -year-old children's counting scores, even though there was more variation in 4 year olds (Mean Score: 6.35, SE: 2.83) than in 6 year olds (Mean Score: 7.75, SE: 1.16). Thus, subsequent analyses were conducted only on the 4 and 6 year olds. First, for probing how these children's counting skills were related to their proportional reasoning skill, the correlational analyses between children's counting scores and average correct percentage were conducted separately for 4 and 6 year olds. The results of these correlational analyses revealed that these correlations were not significant for either age groups (4 year: $r = -.15$, $p = .54$, 6 year: $r = -.17$, $p = .48$). Second, to determine whether children's higher proficiency in conventional counting skills were related to a more frequent use of the absolute number strategy, additional correlation analyses between children's counting scores and the percentages of children's responses to the absolute number foil were conducted separately for 4 and 6 year olds. The results of these analyses reveal a significant positive relation between 4-year-old children's counting scores and the percentages of their children's responses to the absolute number foil ($r = .64$, $p < .01$). This relationship was not significant for 6 year olds, probably due to the lack of variance in their counting scores (Standard deviation: 0.58). These relations are graphically presented in Figure 4 and Figure 5 respectively for 4 and 6 year olds.

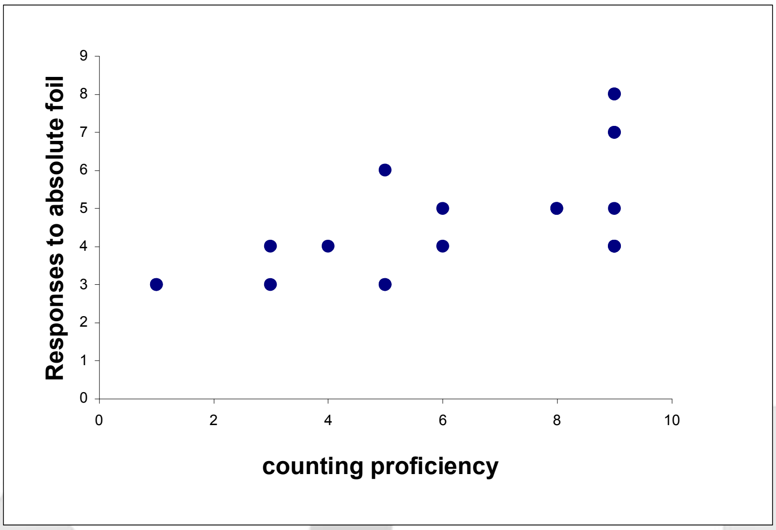

Figure 4. Relation between Counting proficiency and Responses to Absolute foils for 4 year olds

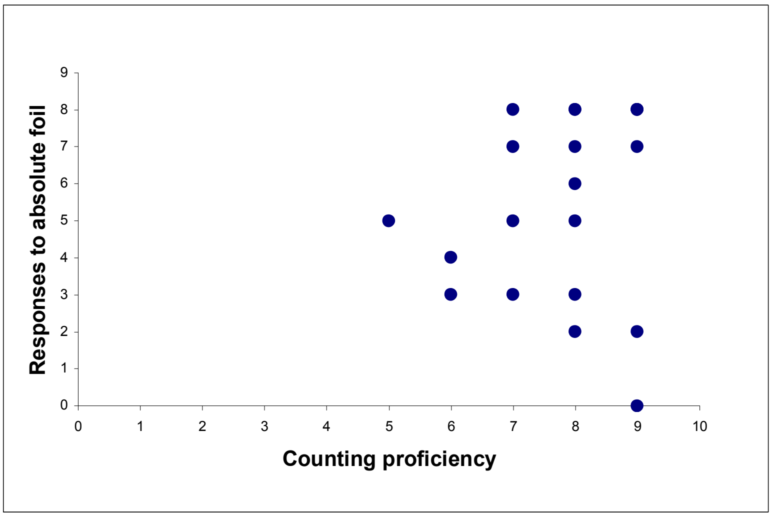

Figure 5. Relation between Counting proficiency and Responses to Absolute foils for 6 year olds

Finally, our results show that the half boundary indeed played an important role in children's matching proportional equivalence, when the proportional tasks were presented in discrete sets. This was supported by the significant main effect of the half boundary (F (3, 228) = 10.15, p< .01). Paired comparisons (Scheffe S, p<. 017, to control for multiple comparisons) revealed that, in all age groups, children performed better when the two foils were crossing the half-boundary (CC) than other categories (CW, WC, WW). To further understand how the half-boundary affected children's responses in the matching task, we again examined children's actual responses in these four different categories regarding the half boundary (CC, CW, WC, WW) for all age groups. When we looked into children's errors without considering the half boundary in the previous analysis, it revealed that young children chose the absolute number foils more frequently. However, when we considered the half-boundary, it was revealed that young children do not always choose the absolute number foil most frequently. Children's response patterns were quite different according to these four categories. Table 3 summarizes the children's responses to problems in each category. When both the foils were crossing the boundary (CC), children never chose the absolute number foil significantly more than chance; 4 and 6 year olds responses to the three choices were evenly distributed and 8 year olds' chose the correct answers significantly more often than chance (t (19) = 3.24, $p < .01$).

Table 3. Average percentage of responses to each choice according to half boundary

Age	Category	Children's response to each choice (Average %)		
		Correct choice	Absolute foil	Random foil
4 Year	CC	37.5	35	27.5
	CW	25	52.5*	22.5
	WC	15	67.5*	17.5
	WW	20	67.5*	12.5
6 Year	CC	35	47.5	17.5
	CW	17.5	67.5*	15
	WC	17.5	67.5*	15
	WW	12.5	70*	17.5
8 Year	CC	75*	20	7.5
	CW	42.5	30	27.5
	WC	40	42.5	17.5
	WW	17.5	40	42.5
10 Year	CC	85*	5	10
	CW	70*	20	10
	WC	70*	20	10
	WW	70*	15	15

*: Children's responses were significantly higher than chance (33%), p <. 05.

IV. Discussion

The current findings lend support to the conclusion that the young children's failures in judging proportions with discrete quantity are related to erroneously use of the number strategy, counting only the number of focal (colored) items without considering the relation between number of focal (colored) and non-focal (uncolored) items. This conclusion was supported by our examination of children's actual responses to the three choices (proportional match, absolute number match, random match); among the three choices, the 4 and 6 year olds responded to the absolute number foil most frequently. This result challenges the previous view that young children completely lack in basic proportional reasoning ability. Children may have failed in several previous studies, because the tasks motivated children to use skill in which they lack, such as calculating portions in a numerically correct way.

As with other kinds of mathematical reasoning (e.g., Siegler, & Taraban, 1986), children may have a variety of possible strategies or ways of thinking about a problem. Likewise, proportional problems can be solved in several different ways. Large variation in young children's performance in previous studies, therefore, may be attributable to different ways of thinking about proportion, one of which is readily available to young children, whereas the other continues to be difficult for them even into late childhood. Our results suggest that the particular strategy that children choose on any given problem is likely to vary with problem characteristic as well as with the child's knowledge. This current study clearly showed that the number strategy is the one that children choose most frequently when the task was presented in a discrete set. 4-and 6-year-old children may fail because they used incorrect number strategy, counting only the number of focal items.

Furthermore, the current study demonstrated that the extent of young children's reliance on such an erroneous counting strategy is closely related to their counting knowledge. The results of our correlational analysis revealed significant relation between 4-year-old children's tendency to use this erroneous number strategy and to their knowledge about natural number. Children with better knowledge about counting convention were more likely to use the number strategy, which was detrimental according to our results. This result indicates that children's choice of a particular strategy depends on their mathematical knowledge as well as the characteristic of problems. Children's strong counting knowledge may have motivated them to count the number of discrete items. Young proficient counters might have been more frequently misled to over-generalize the meaning of natural numbers in the situation where numbers are to be used in a different way. Thus, it is possible that smart children seem to be foolish enough to judge only on the basis of absolute quantity for judging proportions in several previous studies involving discrete quantities.

These results also indicate that one of the confusions in the way young children think about proportion can be traced in part to treating the components of the proportion as separate numbers rather than considering the value of the proportion as a whole (Mack, 1995). This is also consistent with other research that established the children's use of counting discrete items whenever they can, once they can count discrete number exactly (Miller, 1984). For example, Miller (1984) asked children from 3 to 8 years of age to divide different kinds of foods between two turtles in equal amount. Children were not successful at distributing equal amounts of food, because they were only concerned about the number of food units they have generated without considering the size of the unit. Counting number looms so large in young children's everyday mathematical thinking.

On the other hand, our oldest children (10 year olds) were quite successful in our proportional matching task, probably because they used correct number strategy. Our 10 year olds were 5th graders and the testing was done at the end of the school year, which means that they have learned the mathematical conventions necessary for this task and had enough opportunity practicing the skills to master it. The 10-year-old-children might have counted the number of colored and total items, composed fractions with these numbers and found a fraction that was equivalent from the choice cards. Therefore, 10 year olds' success may be mainly related to their mathematical knowledge about representing and comparing proportions. This suggestion may also indicate that if children learn and practice the appropriate mathematical skill, even children younger than 10 year olds may be able to successfully perform in our proportional reasoning task involving discrete quantities. A further study that examines the proportional reasoning skill of Korean children who learn the mathematical skill (fraction) earlier than American children may provide important information about this question. If Korean children's performance at the end of their third elementary school year were better than those of third graders in America, this may provide another evidence that children's failure is attributable to lack of knowledge about mathematical convention rather than to the absence of basic concept about proportions.

Finally, the consideration of the effect of half-boundary allowed us more detailed understanding about children's proportional understanding. Problems based on "half" were solved more successfully than problem where the use of half boundary was not available for children in all age groups. Children performed best when both foils were crossing the boundary (CC). These results are consistent with Spinillo and Bryant's study (1991) about the importance of the half boundary in children's proportional judgments.[1] These findings suggest that "half" plays a general and important role in children's proportional judgments. More importantly, the result shows that young children did not consistently use the number strategy even when proportions are presented in a discrete set. Children in any age group did not choose the absolute number foil significantly more than chance, when the two foils were crossing the half boundary (CC). Thus, when the proportional difference of correct choice from foils was clearly conceivable on the basis of half-boundary, children did not count the number of colored items. Rather, children relied on this basic proportional category for working out proportions. The original argument made by Spinillo and Bryant (1991) regarding this basic skill was that young children lack the ability to reason about the part-whole relation. They hypothesized that young children can reason part-part relations, such as "bigger

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[1] This result is also important in other aspect. Young children's misuse of the number strategy may indicate that children did not define our task as proportional at all. It may indicate that children might have defined our task only as a counting task. However, our results that young children heavily relied on half boundary which is an exclusively relative concept indicate that our children did not always define the task as counting.

than" or "smaller than," but that young children have trouble reasoning the part-whole relation and they argued that higher performance on problems where two choices crossed the half boundary (e.g., 3/8 vs. 5/8) might be attributable to the availability of part-part reasoning (i.e., the colored part is bigger than the uncolored part). In this presents study, there were some verbal and behavioral indications that led us to speculate that young children may have relied on part-part reasoning rather than part-whole reasoning. First, children's explanations about their judgment revealed that they relied on part-part reasoning. A few children voluntarily thought out, for example, that "this one (pointing to the target card) has a big red thing and a small white thing." Additionally, young children's pointing to the colored and uncolored region separately may indicate that they relied more on part-part reasoning.

In conclusion, this current study have demonstrated, using our proportional matching task, that children's performance in judging proportion with discrete quantities was mainly related to their correct/incorrect use of the number strategy, even though this tendency was weakened by children's use of the half-boundary. These results provide important information for resolving the previous contrasting views about development of proportional reasoning ability. Children's success or failure may depend on the type of skills/strategy that the task motivated children to use. Current findings clearly show that tasks involving discrete quantities lead children to rely on a skill that use number information, which young children lack in. A remaining question is what type of strategy even young children can use successfully for judging proportions. A further study examining proportional reasoning ability that does not provide number information may be necessary for deeper understanding about early development of proportional reasoning ability.

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